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ECONOMIC ORDER QUANTITY (EOQ) MODEL WHEN THE DEMAND IS PRICE AND TIME SENSITIVE USING PRESERVATION TECHNIQUES

Article Particulars

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Abstract

Many products cannot be sold after maximum lifetime or expiration dates due to their continuous deterioration. All deteriorating items have their expiration dates. Deterioration can be controlled by spending some cost of item preservation for seasonal products. The market price induceness analyzed with exponentially declining demand. New these days most of the daily consumable items deteriorate over time due to expiration, spoilage, damage etc. The main objective is to maintain the freshness of the products using preservation technology so that it can be used for original purpose. This paper deals with an EOQ model for damageable products when the demand rate is a function of price and time sensitive. Mathematical model is provided to find optimal cycle time and price which maximizes the total profit. Numerical example and sensitive analysis are given to illustrate the theoretical results.

Keywords: *Preservation, deterioration; price- sensitive demand; inventory, declining demand.*

Introduction

Large number of research papers has published on inventory models for deteriorating items such as past foods, green vegetables, bread, blood bank, medicine and fashion goods. Ghare and Schrader [1] considered an EOQ model for exponentially decaying items. Misra [2] established an economic order quantity model with a Weibull deterioration rate for the perishable items. Covert and Philip [3] have studied Weibull distribution and Gamma distribution as deterioration. Number of literature review on EOQ with deteriorating items is done by Raafat [4], Shah and Shah [5], Soni [6], Giri & Chaudhuri [7], Philip [8], Tadikamalla [9], Shah [10], Shah *et al.* [11], Taleizadeh *et al* [12], Teng *et al*. [13], Khanra *et al*. [14], Chen & Teng [15], Sarkar and Sarkar [16], Ghiami & Willims [17], Kaur *et al* [18], Tripathi [19, 20].

Wu *et al*. [21] proposed an economic order quantity model for the retailer where: (i) the supplier provides an up- stream trade credit and the retailer also offers a down-stream trade credit, (ii) the retailer's down- stream trade credit and the buyer not only increases sales revenue but also opportunity cost and default risk, and (iii) deteriorating products not only deteriorate continuously but also have their expiration dates. Dye [22] pointed-out the effect of technology investment on spoilage items. Wee & Widyadana [23] developed a model with stochastic deterioration under preventive maintenance time and rework. Wu et al [24] established a model in which supplier-retailer- customer chain system so that the retailer gets an upstream full trade credit from the supplier whereas offers a downstream partial trade credit to credit – risk customers, the deterioration rate is non- decreasing over time and nearly perfectly close to its expiration. Liu *et al*. [25] constructed an integrated production inventory and preventive maintenance model, which is characterized by the delay- time concept. Wang *et al.* [26] proposed an EOQ model for a seller with the following facts: (i) deteriorating products not only deteriorate continuously but also have their maximum lifetime, and (ii) credit period increases not only demand but also default risk. The remainder of the paper is designed as follows. Section 2 defines notation and assumptions. Section 3 derives the mathematical model followed by optimal solution. Section 5 presents numerical example followed by sensitivity analysis. In the last section concluding remarks are discussed.

Notation and Assumptions

Notations

$I(t)$ inventory level at any instant 't'

K replenishment cost/ order

h unit holding cost /unit time

p unit selling price, $p > c$

c purchase cost/ unit

Θ constant deterioration rate

u preservation technology investment/ unit time to reduce the deterioration rate

$\Phi(u)$ proportion of reduced deterioration rate where $0 \le \Phi(u) \le 1$

Q order quantity

T cycle time

R sales revenue

C_p purchase cost

C_h holding cost per cycle time

I_0 purchase cost

C_0 replenishment cost

$Z(p,T)$ total profit/ cycle time

Assumption

1. Demand rate of the item considered as the function of selling price and present time
2. i.e. *D(p,t) = ap*$^{-β}$.*t* , where a > 0, β > 1.
3. There is no replenishment during the period under consideration.
4. Lead time in negligible.
5. Shortages are not allowed.
6. Preservation technique is considered.
7. (vi)The model is considered for single item.
8. (vii).The proportion of reduced deterioration rate *Φ(u)* is assumed to be continuous and
9. *Φ'(u) > 0, Φ''(u) < 0*.

Mathematical Formulation

Let us consider an inventory consists of constant deterioration rate ϴ with price and time sensitive demand rate. The amount of reduced deterioration rate *Φ(u)* exists when the inventory invests the preservation cost u. The rate of change of inventory with respect to time can described as follows:

$$\frac{dI(t)}{dt} + {}^{-}(1-\text{¤}(u))I(t) = -\sim p^{-}\ t \quad , 0 \le t \le T \qquad (1)$$

under the condition *I(T)* = 0.

The solution of (1) under the above condition is

$$I(t) = \frac{\sim p^{-}}{{}^{-2}\{1-\text{¤}(u)\}^2}\left[\{{}^{-}(1-\text{¤}(u))T-1\}e^{{}^{-}(1-\text{¤}(u))(T-t)} - \{{}^{-}(1-\text{¤}(u))t-1\}\right] \qquad (2)$$

Now

$$Q = I(0) = \frac{\sim p^{-}}{{}^{-2}(1-\text{¤}(u))^2}[\{{}^{-}(1-\text{¤}(u))T-1\}e^{{}^{-}(1-\text{¤}(u))T} + 1] \qquad (3)$$

Sales revenue: The total revenue in time T is

$$R = p\int_0^T (Demand)dt = \frac{\sim p^{1-}\ T^2}{2} \qquad (4)$$

Purchasing cost: The total purchasing cost is

$$C_p = c.Q = \frac{c\sim p^{-}}{{}^{-2}(1-\text{¤}(u))^2}\left[\{{}^{-}(1-\text{¤}(u))T-1\}e^{{}^{-}(1-\text{¤}(u))T} + 1\right] \qquad (5)$$

Holding cost: The formulation for inventory cost is

$$C_h = h\int_0^T I(t)dt = \frac{h\sim p^{-}}{{}^{-2}(1-\ddot{}(u))^2}\left[\left\{\frac{{}^{-}(1-\text{¤}(u))T-1}{{}^{-}(1-\text{¤}(u))}\right\}\{e^{{}^{-}(1-\text{¤}(u))T} - 1\} - T\left\{{}^{-}(1-\text{¤}(u))\frac{T}{2} - 1\right\}\right] \qquad (6)$$

Preservation cost: The preservation cost is $I_0 = uT$ (7)

Replenishment cost: The replenishment cost is $C_0 = K$ (8)

The total profit per unit cycle time can be formulated as

$Z(p,T) = (R - C_p - C_h - I_0 - C_0)/T$

$$= \frac{\sim p^{(1- \ )}T}{2} - \frac{c\sim p^{-}}{T^{-2}\{1-¤(u)\}^2}\left[\{^{-}(1-¤(u))T-1\}e^{^{-}(1-¤(u))T}+1\right] -$$

$$\frac{h\sim p^{-}}{T^{-2}\{1-¤(u)\}^2}\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\{e^{^{-}(1-¤(u))T}-1\} - T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\right] - u - \frac{K}{T} \quad (9)$$

Now, differentiating (9) partially w.r.t. *p* and *T* two times, we get

$$\frac{\partial Z}{\partial p} = \frac{\sim(1- \ )p^{-}\ T}{2} + \frac{c\sim \ p^{-(1+ \ )}}{T^{-2}\{1-¤(u)\}^2}\left[\{^{-}(1-¤(u))T-1\}e^{^{-}(1-¤(u))T}+1\right]$$

$$+\frac{h\sim \ p^{-(1+ \ )}}{T^{-2}\{1-¤(u)\}^2}\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\{e^{^{-}(1-¤(u))T}-1\} - T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\right] \quad (10)$$

$$\frac{\partial^2 Z}{\partial p^2} = \frac{-\sim \ (1- \ )p^{-(1+ \ )}T}{2} - \frac{c\sim \ (1+ \ )p^{-(2+ \ )}}{T^{-2}(1-¤(u))^2}[\{^{-}(1-¤(u))T-1\}e^{^{-}(1-¤(u))T}+1]$$

$$-\frac{h\sim \ (1+ \ )p^{-(2+ \ )}}{T^{-2}\{1-¤(u)\}^2}\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\{e^{^{-}(1-¤(u))T}-1\} - T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\right] \quad (11)$$

$$\frac{\partial Z}{\partial T} = \frac{\sim p^{1-}}{2} + \frac{c\sim p^{-}}{T^{2-2}\{1-¤(u)\}^2}\left[\{^{-}(1-¤(u))T-1\}e^{^{-}(1-¤(u))T}+1\right] - c\sim p^{-}\ e^{^{-}(1-¤(u))T}$$

$$+\frac{h\sim p^{-}}{T^{2-2}\{1-¤(u)\}^2}\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\{e^{^{-}(1-¤(u))T}-1\} - T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\right]$$

$$-\frac{h\sim p^{-}}{^{-}(1-¤(u))}\{e^{^{-}(1-¤(u))T}-1\} + \frac{K}{T^2} \quad (12)$$

$$\frac{\partial^2 Z}{\partial p \partial T} = \frac{\sim(1- \ )p^{-}}{2} - \frac{c\sim \ p^{-(1+ \ )}}{T^{2-2}\{1-¤(u)\}^2}\left[\{^{-}(1-(¤(u))T-1\}e^{^{-}(1-¤(u))T}+1\right] +$$

$$c\sim \ p^{-(1+ \ )}e^{^{-}(1-¤(u))T} - \frac{h\sim \ p^{-(1+ \ )}}{T^{2-2}\{1-¤(u)\}^2}\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\{e^{^{-}(1-¤(u))T}-1\} -\right.$$

$$T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\Bigg]\Bigg] + \frac{h\sim \ p^{-(1+\ )}}{^{-}\{(1-¤(u)\}}\left\{e^{^{-}(1-¤(u))T}-1\right\} \quad (13)$$

$$\frac{\partial^2 Z}{\partial T^2} = -\frac{2c\sim p^{-}}{T^{3-2}\{(1-¤(u)\}^2}\left[\left\{^{-}(1-¤(u))T-1\right\}e^{^{-}(1-¤(u))T}+1\right] - c\sim p^{-}\ e^{^{-}(1-¤(u))T}\left\{^{-}(1-¤(u))-\frac{1}{T}\right\}$$

$$-\frac{2h\sim p^{-}}{T^{3-2}\{(1-¤(u)\}^2}\left[\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\left\{e^{^{-}(1-¤(u))T}-1\right\} - T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\right]\right]$$

$$+\frac{h\sim p^{-}}{T^{-}(1-¤(u))}\left\{e^{^{-}(1-¤(u))T}-1\right\} - h\sim p^{-}\ e^{^{-}(1-¤(u))T} - \frac{2K}{T^3} \quad (14)$$

The optimal values of $p = p^*$ and $T = T^*$ is obtained by solving $\frac{\partial Z}{\partial p} = 0$ and $\frac{\partial Z}{\partial T} = 0$, simultaneously, which maximizes $Z(p,T) = Z(p^*,T^*)$, provided $\frac{\partial^2 Z}{\partial p^2} < 0$, $\frac{\partial^2 Z}{\partial T^2} < 0$ and

$$\left(\frac{\partial^2 Z}{\partial p^2}\right)\left(\frac{\partial^2 Z}{\partial T^2}\right) - \left(\frac{\partial^2 Z}{\partial p \partial T}\right)^2 > 0.$$

$$\frac{\partial Z}{\partial p} = 0 \Rightarrow \frac{(1-\ )pT}{2} + \frac{c}{T^{-2}\{1-¤(u)\}^2}\left[\left\{^{-}(1-¤(u))T-1\right\}e^{^{-}(1-¤(u))T}+1\right]$$

$$+\frac{h}{T^{-2}\{1-¤(u)\}^2}\left[\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\left\{e^{^{-}(1-¤(u))T}-1\right\} - T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\right]\right] = 0$$

(15)

$$\frac{\partial Z}{\partial T} = 0 \Rightarrow \frac{\sim p^{1-}}{2} + \frac{c\sim p^{-}}{T^{2-2}\{1-¤(u)\}^2}\left[\left\{^{-}(1-¤(u))T-1\right\}e^{^{-}(1-¤(u))T}+1\right] - c\sim p^{-}\ e^{^{-}(1-¤(u))T}$$

$$+\frac{h\sim p^{-}}{T^{2-2}\{1-¤(u)\}^2}\left[\left[\left\{\frac{^{-}(1-¤(u))T-1}{^{-}(1-¤(u))}\right\}\left\{e^{^{-}(1-¤(u))T}-1\right\} - T\left\{^{-}(1-¤(u))\frac{T}{2}-1\right\}\right]\right]$$

$$-\frac{h\sim p^{-}}{^{-}(1-¤(u))}\left\{e^{^{-}(1-¤(u))T}-1\right\} + \frac{K}{T^2} = 0 \quad (16)$$

Solving (15) and (16) simultaneously for p and T, we get optimal values p^* and T^*.

Numerical Example

Let us consider the reduced deterioration $f(u) = 1- e^{-\gamma u}$. The other parameter values of the inventory system are taken as follows: a = 500; β = 2; γ = 0.05; c = 5 / unit, h = \$1/ unit/unit time; θ = 0.01; u = 10/ unit time and K = 100. Substituting these values in (15) and (16) and solving for p and T, we get $p^* = 100.197$ and $T^* = 57.41$ years and corresponding $Q^* = 103.863$ units and $Z^*(p,T) = \$ 59.8799$.

Sensitivity Analysis

Sensitivity analysis is the important part of all type business. In any type of business most of the parameters is changed with the change of situation. For example the demand of hot cloths increases in winter season etc. In this section, we check the variation in Q^* and Z^* with the variation in a, h and γ, keeping remaining parameters are same as in numerical example 1.

Table 1 (a): Variation of Q* and Z* with *a*.

A	*p**	*T**	*Q**	*Z*(in dollars)*
600	98.6502	56.5433	124.274	74.2069
700	97.5324	55.9146	144.670	88.5378
800	96.6865	55.4376	165.057	102.871
900	96.0238	55.0632	185.438	117.206

Table 1 (b): Variation of Q* and Z* with *h*.

h	*p**	*T**	*Q**	*Z*(in dollars)*
2	163.267	50.3494	29.2186	26.5624
3	241.034	50.7968	13.6706	14.3745
4	334.866	53.3749	7.90403	8.05046
5	444.209	56.7037	5.14007	4.19283

Table 1 (c): Variation of Q* and Z* with γ.

γ	*p**	*T**	*Q**	*Z*(in dollars)*
0.05	100.197	57.4100	103.863	59.8799
0.10	122.856	74.7955	111.549	64.7635
0.15	152.267	79.1484	117.739	68.7221
0.20	190.262	125.869	122.671	71.8999

From the above Tables the following inferences can be made:

(i) Q^* and Z^* both will increase with the increase of a.
(ii) Q^* and Z^* both will decrease with the increase of h.
(iii) Q^* decreases and Z^* increases with the increase of γ.

Conclusion

In this paper, we have developed an inventory model for deteriorating products using preservation technology. Demand rate is considered to be function of time and exponentially decreasing with price. Preservation cost depends on the situation and place of the product. We have shown that there exists unique optimal selling price and optimal cycle time that maximizes the total profit for the fixed preservation technology interest earned. Numerical examplesis presents to validate the theoretical results. Sensitivity analysis is also provided with the change of important parameters.

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