NOTES ON INTUITIONISTIC FUZZY GRAPHS

[1]Dr.C.Subramani, [2]S.Ramachandran [3]Dr.K.Arjunan & M.Subha
[1]Department of Mathematics, G.T.N. Arts College, G.T.N.nagar, Dindigul, Tamilnadu, India
[2]Department of Mathematics, H.H.The Rajahs College, Pudukkottai, Tamilnadu, India
[3]Department of Mathematics, Easwari Engineering College, Chennai, Tamilnadu, India

Abstract

In this paper, we study some of the properties of various types of degrees, order and size of intuitionistic fuzzy graphs and prove some results on these. Some interesting properties of (α, β)–cut of intuitionistic fuzzy graphs are discussed and some properties of intuitionistic fuzzy regular graphs are also discussed. So we introduce a new structure of a intuitionistic fuzzy graph. 2010Mathematics subject classification: 03E72, 03F55, 05C72

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Introduction

In 1986, Atanassov proposed intuitionistic fuzzy Set (IFS) [3] which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. Atanassov [4] introduced the concept of intuitionistic fuzzy relations and Intuitionistic Fuzzy Graph (IFG). Research on the theory of intuitionistic fuzzy sets has been witnessing an exponential growth in mathematics and its applications. This ranges from traditional mathematics to information sciences. This leads to consider intuitionistic fuzzy graphs and their applications. In this paper we introduce the new structure of Intuitionistic fuzzy graph (IFG).

Preliminaries

Definition

An **intuitionistic fuzzy set** (**IFS**) A of a set X is defined as an object of the form $A = \{(x, \mu_A(x), \gamma_A(x)) \ / \ x\in X\}$, where $\mu_A : X \to [0, 1]$ and $\gamma_A : X\to [0,1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X satisfying $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

Example: Let X = {a, b, c} be a set. Then A = {(a, 0.2, 0.4), (b, 0.1, 0.7), (c, 0.5, 0.3)} is an intuitionistic fuzzy subset of X.

Definition

Let $A = \{(x, \mu_A(x), \gamma_A(x)) / x\in X\}$ and $B = \{(x, \mu_B(x), \gamma_B(x)) / x\in X\}$ be any two intuitionistic fuzzy subsets of X. We define the following

Relations and Operations

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_B(x) \le \gamma_A(x)$ for all x in X.
(ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ for all x in X.

(iii) $A \cap B = \{ (x, \min \{\mu_A(x), \mu_B(x) \}, \max \{\gamma_A(x), \gamma_B(x) \}) / \text{for all } x \in X \}$.
(iv) $A \cup B = \{ (x, \max \{\mu_A(x), \mu_B(x) \}, \min \{\gamma_A(x), \gamma_B(x) \}) / \text{for all } x \in X \}$.
(v) $A^C = \{ (x, \gamma_A(x), \mu_A(x)) / x \in X \}$

Definition

Let $A =\{ (x, \mu_A(x), \gamma_A(x)) / x \in S \}$ be an intuitionistic fuzzy subset of a set S, the strongest intuitionistic fuzzy relation on S, that is an intuitionistic fuzzy relation V with respect to A given by $\mu_V(x, y) = \min \{ \mu_A (x), \mu_A(y) \}$ and $\gamma_V(x,y) = \max\{ \gamma_A (x), \gamma_A(y) \}$ for all x and y in S.

Definition

Let V be any nonempty set, E be any set and $f : E \to V \times V$ be any function. Then A is an intuitionistic fuzzy subset of V, S is an intuitionistic fuzzy relation on V with respect to A and B is an intuitionistic fuzzy subset of E such that $\mu_B(e) \leq \underset{e \in f^{-1}(x,y)}{\mu_s(x,y)}$ and $\gamma_B(e) \geq \underset{e \in f^{-1}(x,y)}{\gamma_s(x,y)}$. Then the ordered triple F = (A, B, f) is called an intuitionistic fuzzy graph, where the elements of A are called intuitionistic fuzzy points or intuitionistic fuzzy vertices and the elements of B are called intuitionistic fuzzy lines or intuitionistic fuzzy edges of the intuitionistic fuzzy graph F. If f(e) = (x, y), then the intuitionistic fuzzy points $(x, \mu_A(x), \gamma_A(x))$, $(y, \mu_A(y), \gamma_A(y))$ are called adjacent intuitionistic fuzzy points and intuitionistic fuzzy points $(x, \mu_A(x), \gamma_A(x))$, intuitionistic fuzzy line $(e, \mu_B(e), \gamma_B(e))$ are called incident with each other. If two distinct intuitionistic fuzzy lines $(e_1, \mu_B(e_1), \gamma_B(e_1))$ and $(e_2, \mu_B(e_2), \gamma_B(e_2))$ are incident with a common intuitionistic fuzzy point, then they are called adjacent intuitionistic fuzzy lines.

Definition

An intuitionistic fuzzy line joining an intuitionistic fuzzy point to itself is called an intuitionistic fuzzy loop.

Definition

Let F = (A, B, f) be an intuitionistic fuzzy graph. If two or more intuitionistic fuzzy edges of Definition F have the same intuitionistic fuzzy vertices, then these intuitionistic fuzzy edges are called intuitionistic fuzzy multiple edges. An intuitionistic fuzzy graph which has intuitionistic fuzzy multiple edges are called an intuitionistic fuzzy multigraph.

Let F = (A, B, f) be an intuitionistic fuzzy graph. If both intuitionistic fuzzy loops and intuitionistic fuzzy multiple edges, then the intuitionistic fuzzy graph F is called an intuitionistic fuzzy pseudo graph.

Definition

An intuitionistic fuzzy graph F = (A, B, f) is called an intuitionistic fuzzy simple graph if it has neither intuitionistic fuzzy multiple edges nor intuitionistic fuzzy loops.

Example

Let $V = \{ v_1, v_2, v_3, v_4, v_5\}$ and E = { a, b, c, d, e, h, g } be two non-empty sets. The function $f : E \to V \times V$ is defined by $f(a) = (v_1, v_2)$, $f(b) = (v_2, v_2)$, $f(c) = (v_2, v_3)$, $f(d) = (v_3, v_4)$, $f(e) = (v_3, v_4)$, $f(h) = (v_4, v_5)$, $f(g) = (v_1, v_5)$. An intuitionistic fuzzy subset $A = \{ (v_1, 0.8, 0.1), (v_2, 0.7, 0.2), (v_3, 0.8, 0.2), (v_4, 0.9, 0.1), (v_5, 0.5, 0.2) \}$ of V, intuitionistic fuzzy relation $S = \{ ((v_1, v_1), 0.8, 0.1), ((v_1, v_2), 0.7, 0.2), ((v_1, v_3), 0.8, 0.2), ((v_1, v_4), 0.8, 0.1), ((v_1, v_5), 0.5, 0.2), ((v_2, v_1), 0.7, 0.2), ((v_2, v_2), 0.7, 0.2), ((v_2, v_3), 0.7, 0.2), ((v_2, v_4), 0.7, 0.2), ((v_2, v_5), 0.5, 0.2), ((v_3, v_1), 0.8, 0.2), ((v_3, v_2), 0.7, 0.2),$

((v_3, v_3), 0.8, 0.2), ((v_3, v_4), 0.8, 0.2), ((v_3, v_5), 0.5, 0.2), ((v_4, v_1), 0.8, 0.1), ((v_4, v_2), 0.7, 0.2),((v_4, v_3), 0.8, 0.2), ((v_4, v_4), 0.9, 0.1), ((v_4, v_5), 0.5, 0.2), ((v_5, v_1), 0.5, 0.2), ((v_5, v_2), 0.5, 0.2), ((v_5, v_3), 0.5, 0.2), ((v_5, v_4), 0.5, 0.2), ((v_5, v_5), 0.5, 0.2) } on V with respect to A and the intuitionistic fuzzy subset B = {(a, 0.7, 0.2), (b, 0.6, 0.2), (c, 0.6, 0.3), (d, 0.7, 0.2), (e, 0.8, 0.2), (h, 0.4, 0.3), (g, 0.3,0 .2) } of E. Then F = (A, B, f) is an intuitionistic fuzzy graph.

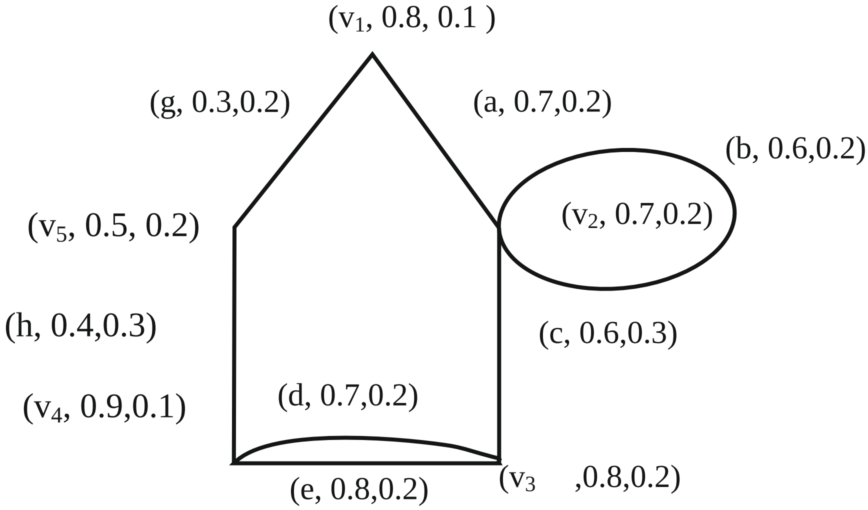

Fig. 1.1

From the Fig. 1.1, (i) (v_1, 0.8, 0.1) is an intuitionistic fuzzy point. (ii) (a, 0.7,0.2) is an intuitionistic fuzzy edge.(iii) (v_1, 0.8, 0.1) and (v_2, 0.7,0.2) are intuitionistic fuzzy adjacent points.(iv) The intuitionistic fuzzy edge (a, 0.7, 0.2) is incident with intutionistic fuzzy vertices (v_1, 0.8,0.1) and (v_2, 0.7,0.2). (v) (a, 0.7,0.2) and (g, 0.3,0.2) are the adjacent intuitionistic fuzzy lines.vi) (b, 0.6,0.2) is an intuitionistic fuzzy loop. (vii) (d, 0.7,0.2) and (e, 0.8,0.2) are intuitionistic fuzzy multiple edges. (viii) It is not an intuitionistic fuzzy simple graph. (ix) It is an intuitionistic fuzzy pseudo graph.

Example

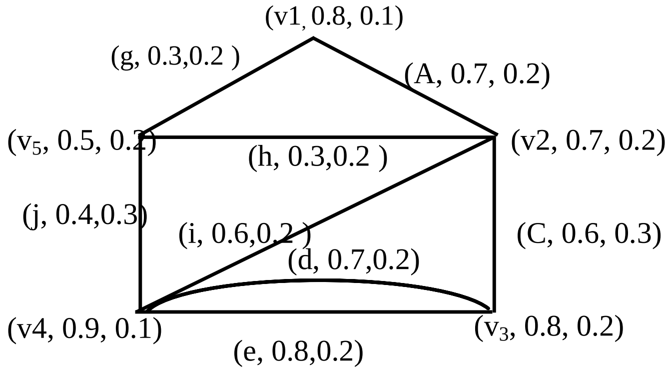

Fig. 1.2 An intuitionistic fuzzy multigraph

Definition

An intuitionistic fuzzy graph H = (C, D, f) where C and D is called an intuitionistic fuzzy subgraph of F = (A, B, f) if $C \subseteq A$ and $D \subseteq B$.

Definition

An intuitionistic fuzzy subgraph H = (C, D, f) is said to be an intuitionistic fuzzy spanning subgraph of F = (A, B, f) if C = A.

Definition

An intuitionistic fuzzy subgraph H = (C, D, f) is said to be an intuitionistic fuzzy induced subgraph of F = (A, B, f) if H is the maximal intuitionistic fuzzy subgraph of F with intuitionistic fuzzy point set C.

Definition

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the sets V and E. Let C be an intuitionistic fuzzy subset of V and $C \subseteq A$. Then intuitionistic fuzzy subset D of E is defined as $\mu_D(e) = \min\{ \mu_C(u), \mu_C(v), \mu_B(e)\}$, $\gamma_D(e) = \max\{ \gamma_C(u), \gamma_C(v), \gamma_B(e) \}$for all e in E. Then H = (C, D, f) is called intuitionistic fuzzy partial subgraph of F.

Definition

Let F = (A, B, f) be an intuitionistic fuzzy graph. Let $(x, \mu_A(x), \gamma_A(x))$ be an intuitionistic fuzzy point of F. The intuitionistic fuzzy subgraph of F obtained by removing the intuitionistic fuzzy point $(x, \mu_A(x), \gamma_A(x))$ and all the intuitionistic fuzzy lines incident with $(x, \mu_A(x), \gamma_A(x))$ is called the intuitionistic fuzzy subgraph obtained by the removal of the intuitionistic fuzzy point $(x, \mu_A(x), \gamma_A(x))$ and is denoted $F- (x, \mu_A(x), \gamma_A(x))$. If $F- (x, \mu_A(x), \gamma_A(x)) = (C, D, f)$ then $C = A - \{ (x, \mu_A(x), \gamma_A(x)) \}$ and $D = \{ (e, \mu_B(e), \gamma_B(e)) / (e, \mu_B(e), \gamma_B(e)) \in B$ and $(x, \mu_A(x), \gamma_A(x))$ is not incident with $(e, \mu_B(e), \gamma_B(e)) \}$. Clearly $F- (x, \mu_A(x), \gamma_A(x))$ is intuitionistic fuzzy induced subgraph of F. If $(e, \mu_B(e), \gamma_B(e)) \in B$ then $F - (e, \mu_B(e), \gamma_B(e)) = (A, D, f)$ is called intuitionistic fuzzy subgraph of F obtained by the removal of the intuitionistic fuzzy line $(e, \mu_B(e), \gamma_B(e))$, where $D = B - \{ (e,\mu_B(e),\gamma_B(e)) \}$. Clearly $F - (e, \mu_B(e), \gamma_B(e))$ is an intuitionistic fuzzy spanning subgraph of F which contains all the lines of F except $(e, \mu_B(e), \gamma_B(e))$.

Definition

From an intuitionistic fuzzy graph F, delete the all intuitionistic fuzzy loops and delete the more than one intuitionistic fuzzy edges from the intuitionistic fuzzy multiple edges, this type of graph is called an intuitionistic fuzzy underling simple graph of F.

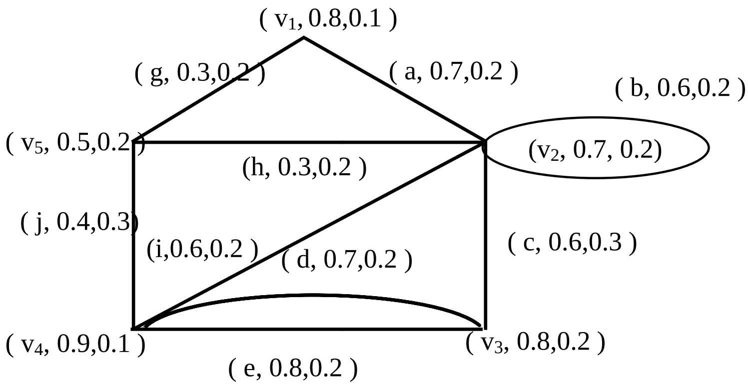

Fig. 1.3 An intuitionistic fuzzy pseudo graph F = (A, B, f)

Example

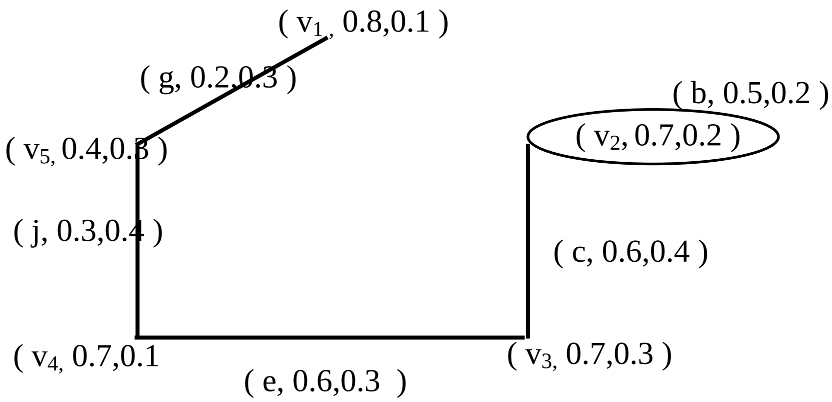

Fig.1.4 An intuitionistic fuzzy subgraph of F

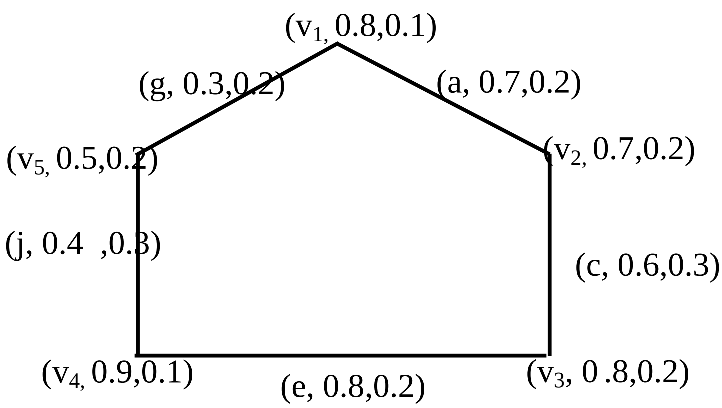

Fig.1.5 An intuitionistic fuzzy spaning subgraph of F

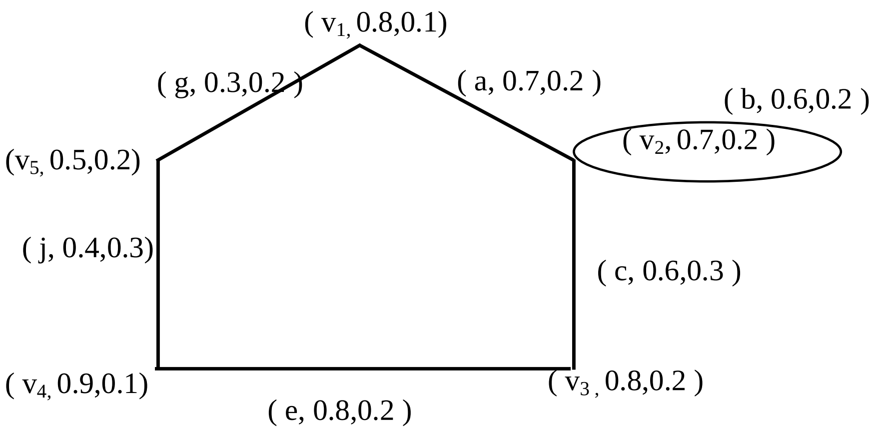

Fig.1.6 An intuitionistic fuzzy subgraph of F induced by P = {v_1, v_3, v_4,v_5}

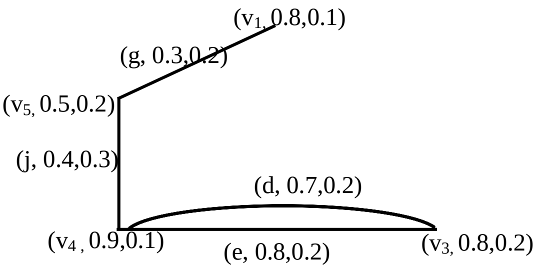

Fig.1.7 An intuitionistic fuzzy partial subgraph of F induced by C where
C={(v_1,0.6, 0.3), (v_3 , 0.6, 0.4) (v_4, 0.8, 0.1), (v_5 , 0.4, 0.2) }

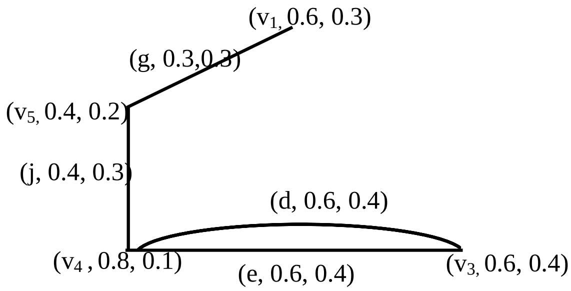

Fig.1.8 F – (v_2, 0.7, 0.2) (h, 0.3, 0.2)

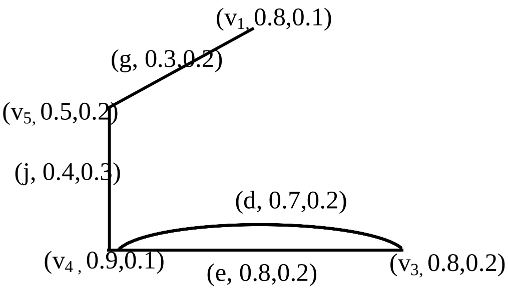

Fig.1.9 F – (b, 0.6) (i,0.6,0.2)

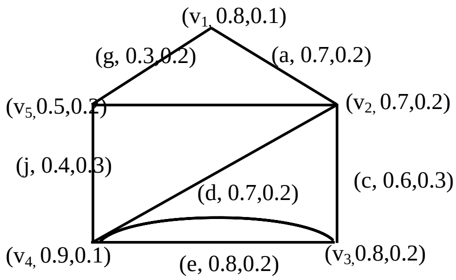

Fig.1.10 An intuitionistic fuzzy underling simple graph of F.

Definition

Let A be an intuitionistic fuzzy subset of X. Then the level subset or α- cut of A is $A_\alpha = \{ x \in X / \mu_A(x) \geq \alpha \text{ and } \gamma_A(x) \leq 1- \alpha \}$, where $\alpha \in [0,1]$.

Definition

Let A be an intuitionistic fuzzy subset of X. Then the strong level subset or strong α- cut of A is $A_{\alpha+} = \{ x \in X / \mu_A(x) > \alpha \text{ and } \gamma_A(x) < 1- \alpha \}$, where $\alpha \in [0,1]$.

Definition

Let A be an intuitionistic fuzzy subset of X. Then (α, β) – cut of A is $A_{\alpha,\beta} = \{x \in X / \mu_A(x) \geq \alpha \text{ and } \gamma_A(x) \leq \beta\}$, where $\alpha, \beta \in [0,1]$.

Definition

Let A be an intuitionistic fuzzy subset of X. Then strong (α, β) – cut of A is $A_{\alpha+,\beta+} = \{x \in X / \mu_A(x) > \alpha \text{ and } \gamma_A(x) < \beta\}$, where $\alpha, \beta \in [0,1]$.

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E and $A_{\alpha,\beta}$ of A and $B_{\alpha,\beta}$ of B. Then $F_{\alpha,\beta} = (A_{\alpha,\beta}, B_{\alpha,\beta}, f)$ is a subgraph of G = (V, E, f).

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E. Let α_1, α_2, β_1, $\beta_2 \in [0,1]$ and $\alpha_1 \geq \alpha_2$, $\beta_1 \leq \beta_2$. Then (A_{α_1,β_1} , B_{α_1,β_1} , f) is a subgraph of (A_{α_2,β_2} , B_{α_2,β_2} , f).

Theorem

Let H = (C, D, f) be an intuitionistic fuzzy subgraph of F=(A, B, f) and $\alpha, \beta \in [0,1]$. Then $H_{\alpha,\beta}$ = ($C_{\alpha,\beta}$, $D_{\alpha,\beta}$, f) is a subgraph of $F_{\alpha,\beta}$ = ($A_{\alpha,\beta}$, $B_{\alpha,\beta}$, f). 1.26 Theorem: Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E and $A_{\alpha+,\beta+}$ of A and $B_{\alpha+,\beta+}$ of B. Then $F_{\alpha+,\beta+}$ = ($A_{\alpha+,\beta+}$, $B_{\alpha+,\beta+}$, f) is a subgraph of G = (V, E, f).

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E. Let α_1, α_2, β_1, $\beta_2 \in [0,1]$ and $\alpha_1 \geq \alpha_2$, $\beta_1 \leq \beta_2$.Then (A_{α_1+,β_1+} , B_{α_1+,β_1+} , f) is a subgraph of (A_{α_2+,β_2+} , B_{α_2+,β_2+} , f).

Theorem

Let H = (C, D, f) be an intuitionistic fuzzy subgraph of F =(A, B, f) and $\alpha, \beta \in [0,1]$. Then $H_{\alpha+,\beta+}$ = ($C_{\alpha+,\beta+}$, $D_{\alpha+,\beta+}$, f) is a subgraph of $F_{\alpha+,\beta+}$ = ($A_{\alpha+,\beta+}$, $B_{\alpha+,\beta+}$, f).

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E and let α_1, α_2, β_1, $\beta_2 \in [0, 1]$. If F_{α_1,β_1} and F_{α_2,β_2} are two subgraphs of G, then (i) $F_{\alpha_1,\beta_1} \cap F_{\alpha_2,\beta_2}$ is a subgraph of G. (ii) $F_{\alpha_1,\beta_1} \cup F_{\alpha_2,\beta_2}$ is a subgraph of G.

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to thelet V and E and let α_1, α_2, β_1, $\beta_2 \in [0, 1]$. If F_{α_1+,β_1+} and F_{α_2+,β_2+} are two subgraphsof G then (i) $F_{\alpha_1+,\beta_1+} \cap F_{\alpha_2+,\beta_2+}$ is a subgraph of G (ii) $F_{\alpha_1+,\beta_1+} \cup F_{\alpha_2+,\beta_2+}$ is a subgraph of G.

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E, the level subsets A_α of A and B_α of B. Then F_α=(A_α, B_α, f) is a subgraph of G = (V, E, f).

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E. Let$\alpha, \beta \in$ [0,1] and $\alpha \leq \beta$ .Then (A_β , B_β , f) is a subgraph of (A_α , B_α ,f). 1.33 Theorem: Let H = (C, D, f) be an intuitionistic fuzzy subgraph of F=(A, B, f) and $\alpha \in [0,1]$. Then H_α= (C_α, D_α, f) is a subgraph of F_α = (A_α, B_α, f). 1.34 Theorem: Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E, the level subsets $A_{\alpha+}$ of A and $B_{\alpha+}$ of B .Then $F_{\alpha+}$ = ($A_{\alpha+}$, $B_{\alpha+}$, f) is a subgraph of G = (V, E, f).

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E.Let $\alpha, \beta \in$ [0,1] and $\alpha \leq \beta$ .Then ($A_{\beta+}$, $B_{\beta+}$,f) is a subgraph of ($A_{\alpha+}$,$B_{\alpha+}$,f).

Theorem

Let H = (C, D, f) be an intuitionistic fuzzy subgraph of F= (A, B, f) and $\alpha \in [0, 1]$. Then $H_{\alpha^+} = (C_{\alpha^+}, D_{\alpha^+}, f)$ is a subgraph of $F_{\alpha^+} = (A_{\alpha^+}, B_{\alpha^+}, f)$.

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E and let α, β∈[0,1]. If F_α and F_β are two subgraphs of G, then (i) $F_\alpha \cap F_\beta$ is a subgraph of G. (ii) $F_\alpha \cup F_\beta$ is a subgraph of G.

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E and α, β∈[0,1]. If $F_{\alpha+}$ and $F_{\beta+}$ are two subgraphs of G, then (i) $F_{\alpha+} \cap F_{\beta+}$ is a subgraph of G. (ii) $F_{\alpha+} \cup F_{\beta+}$ is a subgraph of G.

Definition

Let F = (A, B, f) be an intuitionistic fuzzy graph. Then the degree of an intuitionistic fuzzy vertex is defined by $d(v) = (d_\mu(v), d_\gamma(v))$ where

$$d_\mu(v) = \sum_{e \in f^{-1}(u,v)} \mu_B(e) + 2 \sum_{e \in f^{-1}(v,v)} \mu_B(e) \text{ and } d_\gamma(v) = \sum_{e \in f^{-1}(u,v)} \gamma_B(e) + 2 \sum_{e \in f^{-1}(v,v)} \gamma_B(e).$$

Definition

The minimum degree of the intuitionistic fuzzy graph F = (A, B, f) is $\delta(F) = (\delta_\mu(F), \delta_\gamma(F))$ where $\delta_\mu(F) = \min\{ d_\mu(v) / v \in V \}$ and $\delta_\gamma(F) = \min\{ d_\gamma(v) / v \in V \}$ and the maximum degree of F is $\Delta(F) = (\Delta_\mu(F), \Delta_\gamma(F))$ where $\Delta_\mu(F) = \max\{ d_\mu(v) / v \in V \}$ and $\Delta_\gamma(F) = \max\{ d_\gamma(v) / v \in V \}$.

Definition

Let F = (A, B, f) be an intuitionistic fuzzy graph. Then the order of intuitionistic fuzzy **graph** F is defined to be $o(F) = (o_\mu(F), o_\gamma(F))$ where $o_\mu(F) = \sum_{v \in V} \mu_A(v)$ and $o_\gamma(F) = \sum_{v \in V} \gamma_A(v)$.

Definition

Let F = (A, B, f) be an intuitionistic fuzzy graph. Then the size of the intuitionistic fuzzy graph F is defined to be $S(F) = (S_\mu(F), S_\gamma(F))$ where $S_\mu(F) = \sum_{e \in f^{-1}(u,v)} \mu_B(e)$ and $S_\gamma(F) = \sum_{e \in f^{-1}(u,v)} \gamma_B(e)$.

Example

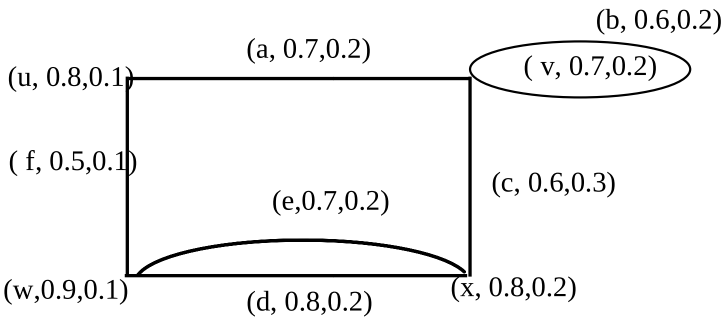

Fig.1.11 Intuitionistic fuzzy graph F

Here d(u) = (1.2, 0.3), d(v) = (2.5, 0.9) , d(w) = (2, 0.5), d(x) = (2.1, 0.7), δ(F) = (1.2, 0.3), Δ(F) = (2.5, 0.9), o(F) = (3.2, 0.6), S(F) = (3.9, 1.2).

Remark

Degree of membership value of intuitionistic fuzzy vertices v need not be equal to the sum of the membership values of all intuitionistic fuzzy edges incident with intuitionistic fuzzy vertex v. Degree of non membership value of intuitionistic fuzzy vertices v need not be equal to the sum of the non membership values of all intuitionistic fuzzy edges incident with intuitionistic fuzzy vertex v.

Theorem

(i) The sum of the degree of membership all intuitionistic fuzzy vertices in an intuitionistic fuzzy graph F = (A, B, f) is equal to twice the sum of the membership value of all intuitionistic fuzzy edges. That is $\sum_{v\in V} d_\mu(v)$ = 2 S_μ (F). (ii) The sum of the degree of non membership all intuitionistic fuzzy vertices in an intuitionistic fuzzy graph F = (A, B, f) is equal to twice the sum of the non membership value of all intuitionistic fuzzy edges. That is $\sum_{v\in V} d_\gamma(v)$ = 2 S_γ (F). (iii) $\sum_{v\in V} d(v) = 2S(F)$.

Proof

(i) Let F = (A, B, f) be an intuitionistic fuzzy graph with respect to the set V and E. Since degree of an intuitionistic fuzzy vertex denote sum of the membership values of all intuitionistic fuzzy edges incident on it, counting each intuitionistic fuzzy loop as twice its membership value. Each intuitionistic fuzzy edge of F is incident with two intuitionistic fuzzy vertices. Hence membership value of each intuitionistic fuzzy edge contributes two to the sum of degrees of intuitionistic fuzzy vertices. Hence the sum of the degree of all intuitionistic fuzzy vertices in an intuitionistic fuzzy graph is equal to twice the sum of the membership value of all intuitionistic fuzzyedges.Thatis $\sum_{v\in V} d_\mu(v)$ = 2 S_μ (F).

(ii) Similarly, $\sum_{v\in V} d_\gamma(v)$ = 2 S_γ (F). (iii) From (i) and (ii) $\sum_{v\in V} d(v) = 2S(F)$.

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy graph with n-intuitionistic fuzzy vertices and all of the intuitionistic fuzzy vertices have degree (s_μ, s_γ) or (t_μ, t_γ). If F has p-intuitionistic fuzzy vertices of degree (s_μ, s_γ) and n–p intuitionistic fuzzy vertices of degree (t_μ, t_γ), then 2S(F) = p(s_μ, s_γ) + (n-p) (t_μ, t_γ).

Proof

Let V_1 be the set of all intuitionistic fuzzy vertices with degree (s_μ, s_γ). Let V_2 be the set of all intuitionistic fuzzy vertices with degree (t_μ, t_γ). Then $\sum_{v\in V} d(v) = \sum_{v\in V_1} d(v) + \sum_{v\in V_2} d(v)$ which implies that 2 S(F) = $\left(\sum_{v\in V_1} d_\mu(v), \sum_{v\in V_1} d_\gamma(v)\right) + \left(\sum_{v\in V_2} d_\mu(v), \sum_{v\in V_2} d_\gamma(v)\right)$ which implies that 2S(F) = p(s_μ, s_γ) + (n-p) (t_μ, t_γ).

Intuitionistic Fuzzy Regular
Definition

An intuitionistic fuzzy graph F = (A, B, f) is called **intuitionistic fuzzy regular graph** if d(v) = (k_1, k_2) for all v in V. It is also called intuitionistic fuzzy (k_1, k_2)- regular graph.

Remark

F is an **intuitionistic fuzzy (k_1, k_2)-regular graph** if and only if δ(F) = Δ(F) = (k_1, k_2).

Example

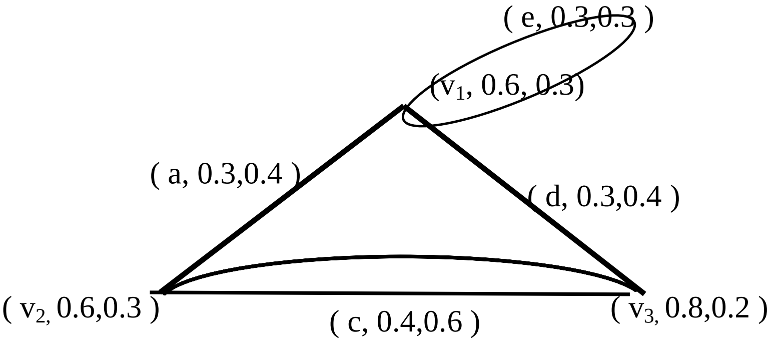

Fig.2.1

Here d(v_1) = (1.2, 1.4) ,d(v_2) = (1.2, 1.4), d(v_3) = (1.2, 1.4), δ(F)=(1.2, 1.4), Δ(F) = (1.2, 1.4). Clearly it is an intuitionistic fuzzy (1.2, 0.6) -regular graph.

Definition

An intuitionistic fuzzy graph F = (A, B, f) is called an **intuitionistic fuzzy complete graph** if every pair of distinct intuitionistic fuzzy vertices are adjacent and $\mu_B(e) = \underset{e\in f^{-1}(x,y)}{\mu_S(x,y)}$ and $\gamma_B(e) = \underset{e\in f^{-1}(x,y)}{\gamma_S(x,y)}$ for all x, y in V.

Example

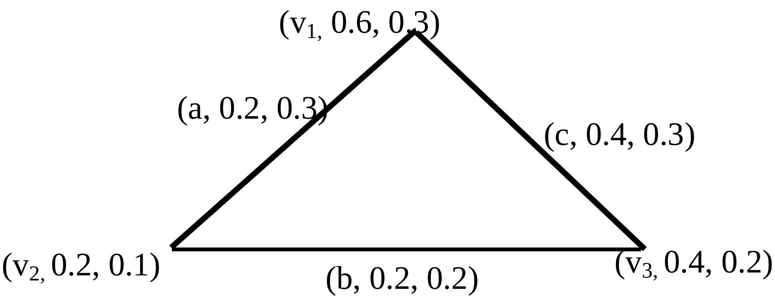

Fig.2.2 Intuitionistic fuzzy complete graph

Definition

An intuitionistic fuzzy graph F = (A, B, f) is an **intuitionistic fuzzy strong graph** if $\mu_B(e) = \underset{e \in f^{-1}(x,y)}{\mu_S(x,y)}$ and $\gamma_B(e) = \underset{e \in f^{-1}(x,y)}{\gamma_S(x,y)}$ for all e in E.

Example

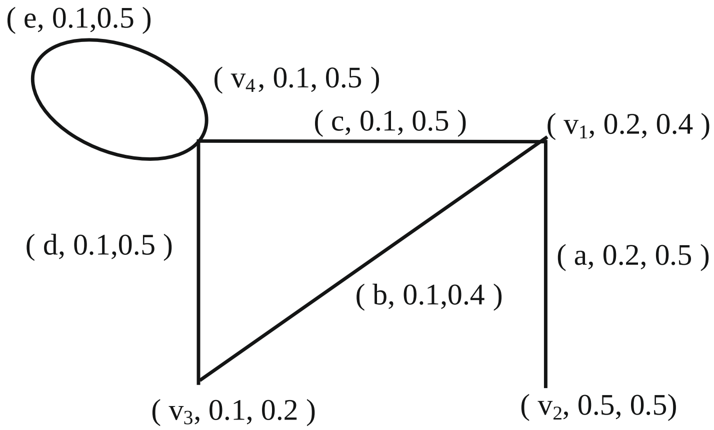

Fig.2.3 Intuitionistic fuzzy strong graph

Remark

Every intuitionistic fuzzy complete graph is an intuitionistic fuzzy strong graph but an intuitionistic fuzzy strong graph need not be intuitionistic fuzzy complete graph from the Fig. 2.3.

Theorem

If F is an intuitionistic fuzzy (k_1, k_2)-regular graph with p- intuitionistic fuzzy vertices then $2S(F) = (pk_1, pk_2)$.

Proof

Given that F is an intuitionistic fuzzy (k_1, k_2)-regular graph with p- intuitionistic fuzzy vertices, so $d(v) = (k_1, k_2)$ for all v in V, $\sum_{v \in V} d(v) = (\sum_{v \in V} d_\mu(v), \sum_{v \in V} d_\gamma(v)) = (\sum_{v \in V} k_1, \sum_{v \in V} k_2)$ $= (pk_1, pk_2)$ which implies that $2S(F) = (pk_1, pk_2)$.

Remark

In a crisp graph theory any complete graph is regular but in this intuitionistic fuzzy graph, every intuitionistic fuzzy complete graph need not be intuitionistic fuzzy regular graph. Consider the example, in Fig. 2.2, it is an intuitionistic fuzzy complete graph but not an intuitionistic fuzzy regular graph since $d(v_1) = (0.6, 0.6)$, $d(v_2) = (0.4, 0.5)$, $d(v_3) = (0.6, 0.5)$.

Theorem

Let F = (A, B, f) be an intuitionistic fuzzy complete graph with p- intuitionistic fuzzy vertices and A be a (k_1, k_2)-constant function. Then F is an intuitionistic fuzzy $((p-1)k_1, (p-1)k_2)$-regular graph.

Proof

Since $A(v) = (k_1, k_2)$ for all v in V and F is an intuitionistic fuzzy complete graph with p- intuitionistic fuzzy vertices and. so $\mu_B(e) = \underset{e\in f^{-1}(x,y)}{\mu_S(x,y)}$ and $\gamma_B(e) = \underset{e\in f^{-1}(x,y)}{\gamma_S(x,y)}$ for all x and y in V. Then $\mu_B(e) = \mu_A(x) \cap \mu_A(y) = k_1$ and $\gamma_B(e) = \gamma_A(x) \cap \gamma_A(y) = k_2$ for all x and y in V. Therefore $d(v) = ((p-1)k_1, (p-1)k_2)$ for all v in V.

Theorem

If F = (A, B, f) is an intuitionistic fuzzy complete graph with p- intuitionistic fuzzy vertices and A is a (k_1, k_2)-constant function then $S(F) = ({}^pC_2 k_1, {}^pC_2 k_2)$ for all v in V.

Proof

Assume F is an intuitionistic fuzzy complete graph with p- intuitionistic fuzzy vertices and $A(v) = (k_1, k_2)$ for all v in V, then$d(v) = ((p-1)k_1, (p-1)k_2)$ for all v in V.

Then $\sum_{v\in V} d(v) = \left(\sum_{v\in V}(p-1)k_1, \sum_{v\in V}(p-1)k_2\right)$ which implies that $2S(F) = (p(p-1)k_1, p(p-1)k_2)$. Hence $S(F) = ({}^pC_2 k_1, {}^pC_2 k_2)$.

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