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Types of Justification in Interdisciplinary Mathematical Modeling Activities

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Abstract
This study aimed to examine the types of justifications presented by 8th-grade middle school students during interdisciplinary mathematical modeling activities. Furthermore, it addresses how these justifications become visible at different stages of the modeling process and in the context of mathematics and science. The research was conducted using a case study design and qualitative research method. The study group consisted of eight 8th-grade students enrolled in the second semester of the 2024–2025 academic year, selected using purposeful sampling. Data were collected from interdisciplinary mathematical modeling activities involving mathematics and science disciplines through video recordings, audio recordings, and students' written outputs. In the data analysis, students' oral and written statements were examined within an argumentation-based framework, and justification components were identified and classified according to the relevant justification typology. The findings showed that students used knowledge-based, interpretive, method-based, observation-based, authority-based, and informal knowledge-based justifications at different stages of the modeling process to justify their answers. It was also found that these types of justification were not limited to a specific stage; however, it was determined that justifications become visible with different functions in the stages of understanding the problem, establishing context, creating a mental model, model building-solving, and evaluation. While justifications in the context of mathematics are mostly related to measurement, calculation, selection of operations, and quantitative model building processes, justifications in the context of science are shaped through concepts such as environmental impact, recycling, energy consumption, carbon emissions, and material properties. The results show that interdisciplinary mathematical modeling activities offer a functional learning environment that makes students' justification processes visible. Accordingly, it is important to design learning environments in modeling activities that support students not only in reaching results but also in justifying solution processes. Considering the limitations of this study, future research should examine the types and functions of justification across different grade levels, disciplinary combinations, and modeling tasks with larger and more diverse participant groups. In addition, using individual interviews, stimulated recall interviews, or other complementary data sources may provide a more comprehensive understanding of students' individual justification processes, alongside those that emerge during group interactions.

Keywords: Interdisciplinary Learning, Mathematical Modeling, Types of Justification, Argumentation, STEM Education, Mathematical Reasoning

Introduction

The multifaceted nature of real-life problems makes it difficult to address them from a single disciplinary perspective. Therefore, in learning environments, it is important to connect knowledge and skills from different disciplines to a specific topic, concept, or problem (Durmuş, 2019). The interdisciplinary approach is considered a program concept based on the holistic use of methods and knowledge from different disciplines, and it contributes to students establishing meaningful connections between lessons and real life (Apostel, 1970; Budak Coşkun, 2009; Jacobs, 1989).

This holistic perspective provides an important foundation for addressing real-world problems in mathematics education. Mathematical modeling encompasses the processes of expressing real-life situations mathematically, transforming them into models considering mathematical properties, and using the developed model in a real-life context (Blum & Niss, 1991; Doerr & Engish, 2003). Mathematical modeling is considered interdisciplinary because it brings together mathematics with real-life situations from different fields, and it is regarded as an approach with the potential to integrate STEM disciplines (Banks & Barlex, 2014; Borromeo Ferri & Mousoulides, 2017; Doğan et al., 2018; English, 2017, 2022; Kertil & Gürel, 2016).

The DMM (Differential Mathematical Modeling) framework suggests that integrating mathematics with at least one STEM field in the mathematical modeling process can create a more comprehensive learning environment. This process is dynamic and requires problem-solving as well as the expression, justification, and persuasion of ideas (Mendonça & Justi, 2013). Therefore, argumentation skills play a significant role in the modeling process; students' explanations of their mathematical ideas, their discussions, and their attempts to persuade each other contribute to the progress of the process (Aydın Güç & Kuleyin, 2021; Tekin Dede, 2019). Argumentation is defined as a structured reasoning process in which individuals defend their claims by supporting them with justifications and provide evidence-based explanations for opposing perspectives (Osborne et al., 2004).

In the literature, the relationship between mathematical modeling and argumentation is mostly addressed through students' problem-solving skills, model development processes, and ways of expressing their mathematical thinking (Aydın Güç & Kuleyin, 2021; Brown & Redmond, 2007; Kuleyin, 2022; Tekin Dede, 2019). While these studies reveal the role of argumentation in the modeling process, they explain, to a limited extent, the foundations of the justifications used by students and the different functions that these justifications assume throughout the modeling stages. Solar et al. (2025) examined the relationship between argumentation and the stages of the mathematical modeling cycle, revealing how students' arguments are shaped throughout the modeling process. However, in this study, students' justifications were not categorized according to the information sources they relied upon. Grisendi and Asenova (2025) focused more directly on the types of justifications and classified the justifications used by high school students in their arguments regarding a specific mathematical concept according to their logical and epistemological properties. However, this study is limited to a specific mathematical concept and does not address the types of justification in relation to the modeling process or interdisciplinary contexts.

Studies in the field of interdisciplinary modeling have primarily focused on students' modeling performance, the skills they use in the process, and teacher opinions (Çavuş- Erdem et al., 2021; Güder & Gürbüz, 2018; Özkaya et al., 2023). Current research has expanded in different directions. Kohen (2025) addressed how mathematical modeling tasks can be structured in a unique scientific-engineering context, while Nugraha et al. (2024) highlighted the fundamental components of interdisciplinary STEM education, such as discipline integration, real-life problems, multiple representations, and collaboration. While these studies offer significant contributions to the structure and design of interdisciplinary modeling environments, they do not directly examine the grounds and types of justifications that students present in these environments. Therefore, research examining the functions of students' justification types at different stages of the modeling process and how they are shaped within the contexts of mathematics and science is limited.

Accordingly, this study aims to examine the types of justifications presented by 8th-grade middle school students during interdisciplinary mathematical modeling activities and to determine how these justifications emerge at different stages of the modeling process and within different disciplinary contexts. In this respect, this study aims to contribute to the literature by presenting the forms of justification that emerge in the modeling process from an interdisciplinary perspective.

Theoretical Framework and Literature

Interdisciplinary Mathematical Modeling (IMM) offers a framework based on the integrated use of at least one of the STEM fields along with mathematics in the mathematical modeling process (Doğan et al., 2019). This framework requires students to apply not only mathematical knowledge but also knowledge and skills from different disciplines related to the problem context when solving real-world problems. In this study, the DMM process is discussed in the context of mathematics and science disciplines; the problems used are structured to include concepts from these two disciplines.

The DMM process begins with understanding and defining the problem in a real-world context. In this process, the individual identifies the necessary variables for the solution, develops assumptions, relates the concepts in the problem to relevant disciplines, and creates mental models. Then, this mental model is transformed into a mathematical structure, and a solution is obtained through appropriate operations. The resulting solution was transferred to a real-life context to evaluate its validity and applicability. Finally, a report is prepared that includes the process and results.

The DMM process, which is covered within the scope of mathematics and science, and the cognitive steps followed by the problem solver in this process are shown in Figure 1.

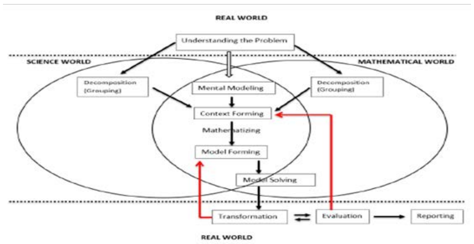

Figure 1 DMM Process in the Context of Mathematics and Science

In the DMM (Diagnostic Modeling) process, it is important for students to reach a solution and understand the reasons behind their solutions. The dynamic nature of the modeling process requires individuals to express their thoughts, justify them, and persuade the opposing side of the argument (Mendonça & Justi, 2013). In this context, argumentation is considered a systematic reasoning process aimed at grounding a particular claim with justifications and providing evidence-based explanations for opposing views (Antonini & Martignone, 2011; Douek, 1999; Osborne et al., 2004).

This study utilized Toulmin's Argumentation Model to identify the justifications presented by students during the modeling process. In Toulmin's (2003) model, the claim refers to the conclusion the argument aims to reach; the data refers to the observations, evidence, and information that form the basis of the claim; and the justification refers to the fundamental structure explaining how the claim was reached from the data. In this context, justification is considered a reasoning process based on presenting valid reasons for accepting a particular claim, thought or statement (Ferreira et al., 2016; Toulmin, 1958). As examining only the structural components of arguments may not be sufficient to explain the nature of justifications, this study utilized the framework developed by Miller et al. (2023) to classify students' justifications in the modeling process.

The relationship between mathematical modeling and argumentation has been addressed from various perspectives in the literature. Brown and Redmond (2007) jointly examined the relationship between argumentation and mathematical applications and student participation, while Tekin Dede (2019) investigated the arguments constructed by prospective teachers during a modeling task within the framework of the modeling cycle and Toulmin's argumentation components. Aydın Güç and Kuleyin (2021) investigated how the quality of argumentation in group modeling processes of sixth-grade students is reflected in their modeling competencies, while Kuleyin (2022) addressed the procedural change in the argumentation skills of eighth-grade students through the quality of argument components. A common aspect of these studies is their focus on the structure, quality, and development of argumentation in modeling processes. However, justifications have mostly been considered a component of the argument structure; the information sources on which justifications are based, the types of justification, and the different functions of these types in the modeling stages have not been systematically compared.

Current research has begun to address the justification and relationship between modeling and argumentation from different perspectives. Harel (2024) considered epistemological justification as a way of thinking related not only to the verification of mathematical knowledge but also to understanding how and why this knowledge was developed. This approach shows that justification not only functions to support a claim but also serves to explain the formation and meaning of the mathematical knowledge. However, Harel's approach explains justification in terms of the epistemological foundations of mathematical knowledge; it does not classify the justifications used by students in the modeling process according to the information sources they rely on. Solar et al. (2025), on the other hand, examined the relationships between students' argumentation processes and the stages of the mathematical modeling cycle, showing that argumentation appears in different forms at different stages of the modeling process. However, in this study, students' justifications were not classified in terms of the information sources they relied on or the types of justification. Grisendi and Asenova (2025) examined the justifications used by high school students in their arguments regarding a specific mathematical concept in terms of their logical and epistemological properties. While this study is significant for its direct focus on the types of justifications, it remains limited to a specific mathematical concept and does not relate justifications to the mathematical modeling process or interdisciplinary contexts. Therefore, although current research addresses justification from various perspectives, studies that examine the sources on which justifications are based, their functions in modeling stages, and their variation according to disciplinary contexts are not sufficiently developed.

In interdisciplinary modeling studies, students' modeling performance, the skills they use in the process, and teacher opinions stand out (Çavuş-Erdem et al., 2021; Güder & Gürbüz, 2018; Özkaya et al., 2023). These studies contribute to the field by demonstrating how knowledge and skills from different disciplines are integrated into the modeling process. However, the detailed basis on which students support their conclusions and how they use mathematical and scientific knowledge sources to justify their findings have not yet been explored.

Current studies also offer new perspectives on the design and structure of interdisciplinary modeling environments. Kohen (2025) examined how mathematical modeling tasks can be structured within a unique scientific-engineering context, proposing a task design that preserves the reality of the interdisciplinary context and supports student participation in the process of mathematical modeling. Nugraha et al. (2024), in their study on interdisciplinary STEM education, identified key components such as disciplinary integration, realistic problems, multiple representations, the engineering design process, student collaboration, and student-centered learning. While these studies provide a strong framework for the design of interdisciplinary learning environments, they have not focused on the types of justifications students produce in these environments and the functions of these justifications in modeling.

In this context, it is necessary to examine not only the existence or structure of the justifications presented by students in the interdisciplinary mathematical modeling process but also their types according to the sources they rely on. Furthermore, revealing the functions of these types of justifications at different stages of the modeling process and how they are shaped within the contexts of mathematics and science can contribute to a more holistic understanding of the relationship between modeling and argumentation.

Purpose of the Study

This study aimed to examine the types of justifications presented by 8th-grade middle school students during interdisciplinary mathematical modeling activities in terms of modeling steps and the contexts of mathematics and science. To this end, the following research questions were addressed.

How do the types of justifications presented by middle school students in the interdisciplinary mathematical modeling process manifest according to the modeling steps?

How do students' types of justifications manifest themselves within the context of mathematics and science disciplines?

Limitations and Assumptions of the Study

This study had some limitations. The study was limited to 8th-grade middle school students who participated in interdisciplinary mathematical modeling activities designed within the context of mathematics and science. Therefore, the findings cannot be directly generalized to different grade levels or modeling activities involving other disciplines. The data were limited to written responses, group discussions, and explanations that students provided during the modeling process. Accordingly, this study focuses on the types of justifications that become apparent through students' oral and written statements. The analyses are limited to the modeling process stages considered and the types of justifications identified.

The study assumes that students express their thoughts honestly during modeling activities and that their written/oral explanations reflect their reasoning processes. Furthermore, it is assumed that interdisciplinary mathematical modeling activities provide a suitable context for students to reveal their types of justification, and that the data collection tools and analysis process are suitable for examining these justifications in terms of the modeling process and disciplinary contexts.

Method

Research Design and Study Group

This qualitative case study aimed to examine the types of justification that emerge in interdisciplinary model-building activities. The case study design was chosen because it allows for a detailed examination of a specific phenomenon in a real-life context (Creswell, 2016). In this research, the case is considered to be the collaborative modeling process of middle school students participating in the model-building process using the disciplines of mathematics and science. The research was conducted with eight 8th-grade students attending a public school in Turkey. Criterion sampling was used to select the study group, aiming to include students with characteristics appropriate to the research objective (Yıldırım and Şimşek, 2008).

In selecting participants, the opinions of mathematics and science teachers were considered, as well as the students' academic standing in both subjects, their potential to establish interdisciplinary relationships, their willingness to participate in group work, and their ability to express their thoughts were taken into account. Focusing on collaborative argumentation processes, the students were divided into two groups of four, each with similar academic characteristics as the other.

Data Collection Tools and Procedure

The data for this study were collected during the second semester of the 2024–2025 academic year. The data collection process involved interdisciplinary model-building activities that required students to develop models using mathematics and science. These activities were selected from modeling problems developed by Doğan et al. (2018), which require relating mathematical and scientific knowledge to real-life contexts. These activities were selected based on their ability to enable students to reason, establish interdisciplinary connections, and engage in group interactions.

Prior to the intervention, warm-up activities, preparatory questions, news articles, short videos, and content based on current events were used to prepare the students for the process. Data were obtained from video and audio recordings, students' written works, reports, drawings, and explanations presented during group work. Video and audio recordings were used to examine group discussions and justification processes, and written works were used to support the oral data. The collected data were organized and prepared for analysis. To support coding consistency, one of the two activities examined in this study was independently coded by a second researcher. The second researcher was provided with a coding framework that included definitions, distinguishing features, and examples of the justification types. Data sections where disagreements arose as a result of comparing the independent codes were evaluated together, considering category definitions and contextual features. A common decision was reached as a result of these discussions. The agreed-upon coding criteria were consistently applied in the analysis of other activities. Basic information regarding the activities used in this study is presented in Table 1.

Table 1 Interdisciplinary Content and Justification Dimensions of the Activities

Activity	Problem Context	Mathematical Dimension	Science Dimension	Mathematical Justification	Science Justification
Plastic Bottle	Recycling, environmental pollution, sustainable living	Estimation, counting, measurement, quantitative reasoning	Recycling, environmental pollution, sustainability	Explaining how calculations and estimations were made; justifying the numerical relationships used	Relating the solution to environmental effects; explaining the contribution of recycling and reuse to nature
Global Warming	Carbon footprint, global warming	Data reading, unit conversion, addition, calculation, proportional reasoning, estimation	Greenhouse gases, carbon emission, environmental impact	Explaining the selection of the data used and the operation steps in the calculations	Relating carbon emissions to environmental effects and grounding the solution recommendations on scientific principles

Data Analysis

In this study, data analysis was conducted to identify the justifications presented by students during the interdisciplinary mathematical modeling activities. Video and audio recordings of small group discussions were transcribed, and students' reports, drawings, and other written materials were included in the analysis. Oral and written data were evaluated together to examine students' explanations and the justifications contained within those explanations.

The analysis was conducted in two stages: In the first stage, the students' oral and written statements were examined within the framework of Toulmin's Argumentation Model to identify the justification components in the arguments. In the second stage, these justifications were classified under the typology developed by Miller et al. (2023) as knowledge-based, observation-based, method-based, interpretation-based, authority-based, and everyday-knowledge-based justification types.

In the coding process, each justification was assigned to the relevant category, considering its source of information, form of explanation, and function. Furthermore, the types of justification were examined according to the stages of the interdisciplinary mathematical modeling process described by Doğan et al. (2019). Thus, the appearance of justification types in the modeling stages and their relationship with the contexts of math and science were evaluated. The definitions and criteria for the justification types used in the analysis are listed in Table 2.

Table 2 Categories and Definitions of Miller et al. (2023) Justification Typology Adapted to This Study

Type of Justification	Definition	Distinguishing Feature
Knowledge-based	Explanations based on students' conceptual or theoretical knowledge	Includes reference to scientific or mathematical knowledge
Observation-based	Explanations based on direct observation, experiment, or experimental results	Includes expressions such as "we observed" or "we tried"
Method-based	Explanations based on the mathematical or scientific methods used in the solution process	Emphasizes procedures, formulas, or methods
Interpretation-based	Explanations based on individual inferences, interpretations, or meaning-making	Includes subjective evaluations
Authority-based	Explanations based on the teacher, textbook, or generally accepted knowledge	Includes expressions such as "the teacher said" or "it is in the book"
Informal knowledge-based	Explanations based on daily life experiences or intuitive knowledge	Includes references to everyday life

Findings

This section presents the findings regarding the types of justifications students exhibited during their interdisciplinary model-building activities. The findings are addressed under two headings, in accordance with the research questions. The first heading examines the appearance of different justification types at various stages of the modeling process, while the second investigates the factors that determine the relationship between these justifications and the contexts of mathematics and science.

The presentation of the findings was based on the modeling steps, and the prominent types of justification were supported with representative examples. This revealed the types of justification students resorted to throughout the process and how these justifications were determined by the disciplinary context. The presentation of the findings includes representative cross-sections reflecting the types of justification prominent in the modeling stages; care has been taken to select examples from different modeling activities.

Examination of Justification Types in the Context of the Modeling Process

This section examines the types of justifications that students present at different stages of the modeling process. The findings were analyzed based on the modeling steps, and the prominent types of justification were explained with representative dialogue examples.

Types of Justification in the Problem Understanding Phase

During the problem comprehension phase, students focused on understanding the problem context and explaining the given situation. At this stage, students not only rephrased the situation presented in the problem text but also developed justifications by establishing cause-and-effect relationships related to the context of the problem. The following dialogue illustrates the process by which students related the concept of carbon footprint to daily life variables during a global warming activity:

Teacher: What is a carbon footprint?

Mercan: It is a measure of the damage caused by human activities to the environment in terms of the amount of carbon dioxide per unit.

Teacher: So, could each of us have a different carbon footprint?

Miraç: Yes. Because everyone uses different amounts of energy in their homes.(justification based on interpretation)

Hayrettin: Unnecessary energy consumption affects our carbon footprint. (justification based on interpretation)

Salih: For example, washing machine usage varies from house to house. (justification based on informal information)

Meryem: Some families may have more than one car. (justification based on informal information)

This dialogue shows that students explained the differences in the carbon footprint of individuals through daily life variables. Miraç and Hayrettin's establishment of a cause-and-effect relationship between energy consumption and carbon footprint is an example of interpretive justification. Salih and Meryem's reliance on everyday life examples, such as washing machine use and car ownership, is considered an example of informal, knowledge-based justification. This finding indicates that in the problem-understanding phase, justifications are mostly shaped by relating the problem context to daily life and providing context-related reasons.

Types of Justification in the Separation and Context Establishment Stages

Following the problem comprehension phase, students began dissecting the concepts involved in the problem and relating them to mathematics and science disciplines. At this stage, students linked concepts such as environmental awareness, recycling, resilience, measurement, and calculation to relevant disciplines. However, some statements remained at the level of concept recognition, and justifications regarding why and how they would be used in problem solving were limited. The following dialogue exemplifies the students' process of establishing interdisciplinary connections through the plastic bottle activity:

Teacher: You said that you would design a playhouse using plastic bottles. Let us consider the disciplines that can be used to solve this problem.

Salih: Science comes first. Because plastic pollutes the environment. (information-based reasoning)

Batuhan: That's why it would be good to reuse plastic bottles. (interpretation-based reasoning)

Teacher: So, which science concepts might be useful? Ali: Recycling… and the durability of plastics.

Teacher: Where does mathematics come into play?

Salih: In determining the dimensions of the playhouse, For example, its height, width…

This dialogue shows the students attempting to identify concepts related to science and mathematics. Salih's statement about plastics polluting the environment is considered an example of fact-based justification, whereas Batuhan's assertion that reusing plastic bottles would be beneficial is considered an example of interpretive justification. Ali's emphasis on recycling and durability, and Salih's reference to the concept of measurement, demonstrate a connection between disciplines; however, since these statements do not contain a clear cause-and-effect relationship, they are not considered justifications.

Similarly, in the global warming activity, students linked the concept of carbon footprint to science and then transitioned to a mathematical process by stating that this concept needed to be calculated. The relationships students established between carbon dioxide levels, energy consumption, and gas emissions were evaluated as knowledge-based justifications; their inferences regarding the need for calculation were evaluated as interpretive justifications; and everyday life examples such as transportation and fuel use were evaluated as informal knowledge-based justifications. This indicates that during the separation and context-building phases, students began to connect science-related concepts with mathematical processes.

Following the dissociation process, students, guided by their teacher, moved on to the context-building process, addressing the problem context in a more holistic manner. The following dialogue serves as an example:

Teacher: Why are we making a playhouse using plastic bottles?

Mercan: Because they don't disappear for a very long time when thrown into nature. (information-based justification)

Zeynep: Yes, the text also emphasized this. (authority-based justification)

Teacher: So, what could be the purpose of doing this project in the schoolyard? Ela: "We are doing it to draw attention to the environment because when it is done in the schoolyard, more people will see it and realize the importance of recycling." (commentary-based justification)

Mustafa: We are both protecting the environment and utilizing waste like plastic bottles because these kinds of materials stay in nature for a long time and when reused, the damage to the environment is reduced. (information-based justification)

This dialogue shows that students associated the use of plastic bottles with environmental impact, visibility, and raising awareness. Mercan and Mustafa's statements are considered fact-based justifications, Zeynep's reference to the text is an authority-based justification, and Ela's linking of the schoolyard with raising awareness is an interpretive justification. This finding indicates that during the context-building phase, students justified their use of concepts in the problem by relating them to real-life purposes.

Types of Justification in the Mental Model Formation Stage

Following the decomposition and context-building phase, students developed assumptions about problem-solving and began discussing possible solutions. At this stage, students explained how they could construct a model by establishing relationships between the identified variables. Below are examples of dialogues reflecting the types of justification that are prominent in the mental model-building process.

In the plastic bottle activity, students identified the structural features of the playhouse and discussed how these features affected the number of plastic bottles that could be used.

Teacher: How will the number of plastic bottles you use affect the doors and windows of your playhouse?

Batuhan: There will be gaps where the doors and windows are. Therefore, we will not use plastic bottles. (justification based on commentary) If we make the roof out of a more durable material instead of plastic bottles, our playhouse will be nicer.

Ali: So we won't be calculating the number of plastic bottles for the doors, windows, and roof. Let us make the roof out of wood. Let us not use plastic bottles. This will be easier for us too. (justification based on commentary)

Teacher: Okay, how many windows do you have?

Batuhan: Let us put four windows. They will sweat while playing. We'll need them. (justification based on informal information)

This dialogue shows that the students created a mental design by considering structural elements, such as doors, windows, and roofs. Their statement that plastic bottles would not be used in areas with doors and windows demonstrates a connection between design decisions and the calculation process and is considered an example of interpretive justification. Batuhan's explanation of the number of windows based on usage needs is an example of informal, fact-based justification.

Types of Justification in Model Creation and Model Deconstruction Phases

During the model building and solving phase, students attempted to produce mathematically and scientifically based solutions using the variables and assumptions they had identified in the previous steps. In the playhouse activity using plastic bottles, students focused on determining the number of plastic bottles required by considering elements such as the dimensions of the structure, doors, windows, and height. In the global warming activity, they structured the solution process to calculate the carbon footprint using variables such as family structure, daily life habits, energy use, and time. At this stage, students were observed to employ different justifications in the processes of measurement, calculation, relating variables, and developing appropriate solution strategies. Examples of student dialogues related to this process are presented below:

Teacher: Friends, How Will you Arrange the Plastic Bottles?

Ela: Let us place the backs outwards; it will look nicer. (Justification based on interpretation)

Teacher: So, how will you determine the number of plastic bottles needed?

Zeynep: Sir, we measured the base diameter of the plastic bottle as 5 cm and its height as 21 cm. (Justification based on observation) For example, suppose one wall of our playroom is 80 cm wide. By dividing 80 by 5, we can determine how many plastic bottles can be placed side by side in a row. (Justification based on method)

Teacher: How will you know how many rows of plastic bottles you need to place on the wall?

Ela: Then, we will consider the height of the wall. By dividing the wall height by 5 cm, we calculated the number of rows of bottles that could be arranged. (Justification based on method)

In this dialogue, Ela's decision about bottle placement reflects interpretive justification because it is based on an aesthetic evaluation. Zeynep's measurement of the bottle dimensions is considered observational justification, whereas the use of division to determine the number of bottles and rows indicates methodological justification. Students' written outputs related to this calculation process are shown in Figure 2.

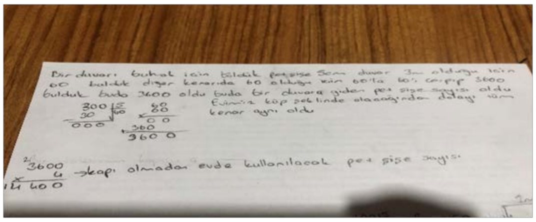

Figure 2 Visual Guide showing the Calculation of the Number of Plastic Bottles that can be Placed on a Wall Surface

In addition to the calculation steps, the students illustrated the models they developed with drawings. Examples of the group model representations are shown in Figure 3.

In the global warming activity, students were also seen to structure a problem-solving process for calculating their carbon footprint. The following dialogue illustrates the types of justifications that students used in this process.

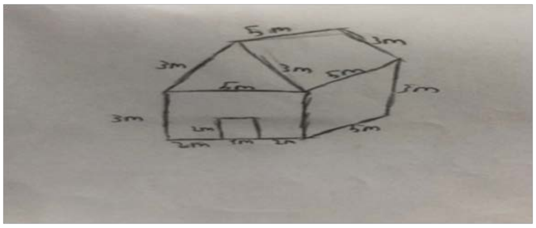

Figure 3 Playhouse Designed in the Shape of a Square Prism

Teacher: How do we calculate our carbon footprint?

Ali: I think we can calculate it by looking at how much a person uses electric vehicles. (justification based on interpretation)

Mirza: To calculate it annually, we need to multiply it by 52. (justification based on method)

Mustafa: A day is 24 hours, right? We'll use that. (justification based on knowledge)

Mirza: Yes, first we multiply 24 by 7, then we multiply by 52 to find the annual value. (justification based on method)

This dialogue shows that students structured the carbon footprint calculation through time units and annualization. Ali's statement reflects interpretive justification, as he relates the calculation to electric vehicle use; Mustafa's use of the 24-hour day is knowledge-based justification; and Mirza's explanation of weekly and annual calculation steps is methodological justification.

Types of Justification During the Conversion and Evaluation Phase

Following the model creation and problem-solving phases, students were observed to transfer their findings to a real-life context, evaluating the applicability of the solution and its contribution to the problem context. Below are examples of dialogues reflecting the types of justification that stand out in the transformation and evaluation processes.

Teacher: How does this playhouse contribute to the environment?

Ali: By using plastic bottles to build the playhouse, we reduced the waste generated because we didn't use normal construction materials. This resulted in less harm to the environment. (justification based on interpretation)

Salih: We have seen that plastic bottles remain in nature for a very long time. It was good that we utilized this waste. (justification based on knowledge)

Batuhan: By using bottles obtained through recycling, we both saved energy and reduced the amount of waste in nature. (justification based on knowledge and interpretation)

This dialogue shows that the students evaluated the playhouse model in terms of its environmental contributions. Ali's statement is interpretative because he established a cause-and-effect relationship between not using construction materials and the reduction in waste; Salih's explanation is considered factual because it is based on the knowledge that plastic bottles remain in nature for a long time. Batuhan's statement, on the other hand, has both factual and interpretative justification characteristics because he used knowledge-based elements such as recycling, energy saving, and waste reduction to interpret the model's environmental contribution.

The analyses show that justification types are not limited to a specific step; they emerge with different functions in the modeling process. Table 3 summarizes the general appearance and functions of these types in the modeling step.

Table 3 Types of Justifications Observed in the Modeling Steps and General Overview

Modeling Stage	Observed Types of Justification	General Overview
Understanding the Problem	Interpretation-based, knowledge-based, informal knowledge-based	Students explained the problem context and developed justifications based on everyday life, environmental impact, and reasons related to the problem situation.
Decomposition and Contextualization	Knowledge-based, interpretation-based, authority-based, informal knowledge-based	Students associated the concepts in the problem with mathematical and scientific knowledge, and supported these relationships through environmental, contextual, and everyday-life-based explanations.

Constructing a Mental Model	Interpretation-based, informal knowledge-based	Students identified the variables that would form the basis of the solution process by developing design decisions, assumptions, and real-life scenarios.
Creating and Solving the Model	Method-based, observation-based, knowledge-based, authority-based, interpretation-based	Students justified measurement, calculation, operation selection, and solution strategies through different bases.
Transforming and Evaluating	Knowledge-based, interpretation-based	Students transferred the results they obtained to real-life contexts and made evaluations based on environmental impact, applicability, and the meaning of the results.

The types of justification presented in Table 3 illustrate examples that become visible at the relevant stages; this does not mean that these types are limited to the stages mentioned. The findings show that students can use the same type of justification at different stages of the modeling process; however, the function of these justifications varies depending on the structure of each stage.

Examination of Justification Types within a Disciplinary Context

Under this heading, the justifications presented by students in the interdisciplinary mathematical modeling process were examined in terms of mathematics and science contexts. The disciplinary context of the justifications was determined by considering the concepts, information, and arguments that the students referred to in their explanations. Accordingly, a general summary of the disciplinary contexts of the justifications in both modeling activities is presented in Table 4.

Table 4 Disciplinary Context of Justifications According to Activities

Activity	Disciplinary Context	Elements Determining the Disciplinary Context
Building a playhouse with plastic bottles	Mathematics	Length measurement; door and window dimensions; calculation of the number of plastic bottles; surface width/height; division
Building a playhouse with plastic bottles	Science	Recycling; environmental pollution; persistence of plastics in nature; reuse of waste materials; material durability
Global warming	Mathematics	Carbon footprint calculation; time units; daily-weekly-yearly conversions; calculation of energy use duration
Global warming	Science	Amount of carbon dioxide; energy consumption; transportation; gas emission; environmental impact of carbon emissions

As shown in Table 4, the disciplinary context of students' justifications is evident through the concepts and arguments they use in their explanations. Justifications within the context of mathematics are related to measurement, calculation, and numerical transformations, whereas justifications within the context of science are shaped by concepts such as environmental impact, recycling, energy consumption, and carbon emissions. This finding reveals the interdisciplinary nature of the justifications that students develop during the modeling process.

Discussion and Conclusion

This study examined the types of justifications presented by 8th-grade middle school students in interdisciplinary mathematical modeling activities. The findings show that justifications have different functions in the modeling stages. Specifically, in the problem understanding and context-building stages, students tended to explain the problem situation, make connections to real life, and identify concepts belonging to related disciplines. This is related to the need for the modeling process to make sense of the real-life context and to make cognitive transitions towards the solution process (Borromeo Ferri & Blum, 2009; Kaiser & Brand, 2015).

During the mental model-building phase, the students focused on the assumptions and variables that formed the basis of the solution process. In the playhouse activity using plastic bottles, design elements stood out, and in the global warming activity, variables such as family structure and daily life habits related to carbon footprint calculation were highlighted. This finding shows that students not only performed calculations in the modeling process but also formed and justified the assumptions on which the solutions were based. This result is consistent with studies that emphasize the nonlinear nature of the modeling process and the transitions between stages (Borromeo Ferri & Blum, 2009; Kaiser & Brand, 2015).

In the model building and solving phase, students' justifications were related to measurement, calculation, and method explanations. Length measurements, division operations, and placement arrangements were used to determine the number of PET bottles, whereas variables such as time units, annual calculations, and energy usage duration were used in the carbon footprint activity. This demonstrates that modeling activities support students not only in applying mathematical operations but also in explaining why and how these operations are used. Therefore, this phase can be considered a process in which method-based justifications are more visible (Doğan et al., 2019; Çavuş- Erdem et al., 2021).

During the transformation and evaluation phase, students assessed the environmental and daily life implications of their solutions by applying their findings to real contexts. In the PET bottle activity, recycling, waste quantity, and environmental contribution were highlighted, and in the global warming activity, carbon footprint, energy consumption, transportation preferences, and daily life habits were emphasized. At this stage, the meaning and applicability of the results in a real-life context were justified, rather than purely mathematical calculations. This finding parallels studies indicating that in interdisciplinary modeling processes, students reinterpret knowledge from different disciplines within the context of the problem (Çavuş-Erdem et al., 2019; Çavuş et al., 2021).

Another finding of this study is that justifications emerge with different foundations in the contexts of mathematics and science. In the context of mathematics, justifications are shaped by measurement, calculation, choice of operations, and quantitative model building processes, while in the context of science, they are shaped by concepts such as environmental pollution, recycling, energy consumption, carbon dioxide levels, and carbon footprints. This situation shows that in interdisciplinary modeling processes, different disciplines support students' ways of thinking and justifying in different ways (Doğan et al., 2019; Çavuş- Erdem et al., 2021).

The findings show that students used different types of justification together in the modeling process. This reveals that the justification component in Toulmin's (1958) argumentation model is important for understanding students' thinking processes; however, justifications should be examined not only in terms of argument structure but also in terms of their types. In this context, the justification typology developed by Miller et al. (2023) provides a detailed functional analytical framework for classifying and interpreting students' justifications in the modeling process.

The theoretical contribution of this study lies in not limiting the treatment of students' justifications solely to a component within Toulmin's argumentation model, but also in classifying these justifications according to their sources and relating them to different stages of the modeling process. In this respect, the study brings together an approach that examines the structure of argumentation with a typological approach focusing on types of justification within the context of interdisciplinary mathematical modeling. The findings show that the same type of justification can assume different functions at different stages of the modeling process, and that justifications are shaped on different grounds in the contexts of math and science. Therefore, it can be said that types of justification can be considered not as fixed categories, but as dynamic structures that acquire functions according to the stages of the modeling process and the disciplinary context.

However, the results obtained must be evaluated within the context of certain limitations. The research is limited to eight 8th-grade students attending a single school and two modeling activities that integrate mathematics and science. Therefore, the findings cannot be directly generalized to different grade levels, student groups, disciplines, or modeling activities in general. Furthermore, because the analyses are based on verbal and written justifications that emerged during group interactions, the students' individual thought processes and justifications not expressed in group discussions could not be fully revealed. In addition, the determination of justification types is limited by the analytical framework used, and different classifications may reveal other dimensions of the same dataset.

Despite these limitations, the study revealed that interdisciplinary mathematical modeling activities offer a powerful learning environment that makes students' justification processes visible. Students developed knowledge-based, interpretive, method-based, observation-based, authority-based, and informal knowledge-based justifications throughout the modeling process. However, it was observed that these types of justifications are not limited to specific modeling steps; they also serve different functions, such as making sense of the problem situation, identifying variables and assumptions, explaining the solution method, and evaluating results in a real-life context. Therefore, students' justifications should be evaluated not only in terms of their types but also in terms of their position, function, and disciplinary basis within the modeling process. In this respect, the study contributes to a more detailed explanation of the relationship between argumentation and mathematical modeling through the types of justification provided.

Suggestions

Considering the limitations of this study, future research should examine students' types of justification at different grade levels and in interdisciplinary mathematical modeling activities that involve disciplines beyond mathematics and science. Such studies may help determine whether students' justification processes vary according to age, prior knowledge, curriculum expectations, or the disciplinary structure of modeling tasks. Given that the present study was conducted with a limited number of participants and focused on specific modeling problems, future studies should include larger and more diverse participant groups and employ a wider range of modeling tasks. This may provide a more comprehensive basis for examining whether similar justification patterns emerge across different classrooms, schools, and educational contexts in the future.

In addition, as the data were limited to students' written responses, group discussions, and oral explanations produced during the modeling process, future studies may incorporate additional data sources, such as individual interviews, stimulated recall interviews, reflective journals, or follow-up tasks. The use of such data sources may allow researchers to explore students' reasoning processes in greater depth and identify justifications that may not be fully visible in group interactions or written products. Furthermore, future research may employ different modeling process frameworks and justification typologies to compare how students' explanations are interpreted from alternative analytical perspectives. Longitudinal studies may also be conducted to investigate how students' justification types develop over time through repeated participation in interdisciplinary mathematical modeling.

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