A Study on Basic Operations and Properties of Fuzzy Matrices and its Sections

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In 1965, L.A.Aadeh introduced the notion of Fuzzy set which deals with the problem of uncertainty in real physical world, since then its application have been grown rapidly in many disciplines.

Fuzzy set theory provides us with a frame work which is wider than that of the classical set therapy. Various mathematical structures whose features emphasize the effects of ordered structure can be developed on the theory.

Fuzzy matrix was defined first time by Thomson in 1977. The theories of fuzzy matrices were developed by kim and Roush as an extension of Boolean matrices [20]. Hemashina et al investigated iterates of fuzzy circulate matrices.

A fuzzy matrix is a matrix with elements having values in the closed interval [0,1].

We deal with many properties of a fuzzy matrix, such as reflexive, reflexive, transitive, nilpotent, regular and others can be extended to all its sections. We also deal with some properties of the sections of a fuzzy matrix do not extend to the original fuzzy matrix such as regularity property.

We also define some properties of a square fuzzy matrix, such as α irrelative, strongly reflexive and circularity, and examine it throughout our results.

Here we also with a concept of trace of a fuzzy matrix has also been investigated in terms of the newly introduced method of fuzzy matrix representation.

Further some of the properties of trace of fuzzy matrix according to the suggested definition have also been proposed.

In case of fuzzy matrices min or max operations are defined in order to get the resulting matrix as a fuzzy matrix, kandasamy [18].

This motivated us to study the notion of fuzzy matrices and sections of fuzzy matrix. In our work we discuss about basic operation of fuzzy matrix, sections of fuzzy matrix and new representation of fuzzy matrix using reference function.

Our dissertation is divided into five chapters. In first chapter we discuss preliminary definitions and needed results which are frequently used elsewhere in the dissertation. In second chapter we deal with some properties of fuzzy matrices. In third chapter we deal with some properties of section of fuzzy matrices. In fourth chapter 4 we deals with new type of representation of fuzzy matrices using reference functions. And fifth chapter contains information about the reference that where referred in making this dissertation.

Preliminaries

In this section, we recall some basic definitions that are required for our work.

Definition

Lex X be a non empty set. A fuzzy set μ of the set X is a function $\mu\ X \to [0,1]$.

Example

X= {1,2,3,4,5}

μ (x) = 1/x for all x εX

Definition

The union two fuzzy sets λ U μ is a fuzzy subset of X defined as (λUμ) (x)) for every x εX.

Example

Let X = {0, 1, 2, 3} be the set.

The fuzzy sets are defined by

$$\mu(X) = \begin{cases} 0.1 & \text{ifx } = 0,1 \\ 0.5 & \text{ifx } = 2 \\ 0.3 & \text{ifx } = 3 \end{cases}$$

$$\mu(x) = \begin{cases} 0.6 & \text{ifx } = 0,1 \\ 0.5 & \text{ifx } = 2 \\ 0.2 & \text{ifx } = 3 \end{cases}$$

Then U μ_2 (x) = max (μ_1 (x), μ_2(x))

$$= \begin{cases} 0.6 & \text{ifx } = 0,1 \\ 0.5 & \text{ifx } = 2 \\ 0.3 & \text{ifx } = 3 \end{cases}$$

Definition

The intersection two fuzzy sets λ and μ of set x, denoted by $\lambda \cap \mu$ is a fuzzy subset of X defined as $(\lambda \cap \mu)$ (x) = min (λ(x), μ(x)) every x εX.

Example

Let X = {0,1,2,3} be the set.

The fuzzy sets are defined by

$$\mu_1(X) = \begin{cases} 0.1 & \text{ifx } = 0,1 \\ 0.5 & \text{ifx } = 2 \\ 0.3 & \text{ifx } = 3 \end{cases}$$

$$\mu_2(x) = \begin{cases} 0.6 & \text{ifx } = 0,1 \\ 0.5 & \text{ifx } = 2 \\ 0.3 & \text{ifx } = 3 \end{cases}$$

Then U μ_1 (x) $\cap \mu_2$(x) = min (μ_1 (x), μ_2(x))

$$= \begin{cases} 0.1 & \text{ifx } = 0,1 \\ 0.5 & \text{ifx } = 2 \\ 0.2 & \text{ifx } = 3 \end{cases}$$

Definition

The complement of a fuzzy set μ of set x, denoted by μ^c and defined as μ^c(x)= 1-μ(x) for every x εX.

Example

X = {0,1,2,3}

$$\mu(x) = \begin{cases} 0.5 \\ 0.2 \\ 0.3 \end{cases}$$

$$\mu^c(x) = \begin{cases} 0.5 \\ 0.2 \\ 0.3 \end{cases}$$

Definition

A rectangular array of numbers is called a matrix. The horizontal arrays a matrix are called its rows and the vertical arrays are called its columns. A matrix having m rows and n columns is said to have the order m x n.

A matrix A of order m x n can be represented in the following form:

$$\left[\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{1n} & \cdots & a_{nn} \end{pmatrix}\right]$$

Example:

$$A = \begin{bmatrix} 1 & 22 & 34 \\ 72 & 67 & 92 \\ 13 & 54 & 81 \end{bmatrix}$$

Definition:

A square matrix A = [aij] with aij = $\begin{cases} 1 & \text{ifi } = j \\ 0 & \text{ifi } \neq j \end{cases}$ is called the identity matrix, denoted by In.

Example:

$$I_n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Definition

The transpose of an m x n matrix A = $[a_{ij}]$ is defined as the n x n matrixB = $[b_{ij}]$, with $b_{ij} = a_{ji}$ for 1 ≤i≤ m and ≤j ≤n. the transpose of A is denoted by A'.

That is, by the transpose of an m x n matrix A, we mean a matrix of order n x m having the rows of A as its columns and the columns of A as its rows.

Example

A = [0.1 0 0.9]

Then transpose is given by

$$A^t = \begin{bmatrix} 0.1 \\ 0 \\ 0.9 \end{bmatrix}$$

Definition

Let A = $[a_{ij}]$ and B = $[b_{ij}]$ be are two m x n matrices. Then the sum A+B is defined to be the matrix C = $[c_{ij}]$ with $c_{ij} = a_{ij} + b_{ij}$.

Example

$$B = \begin{bmatrix} 4 & 1 & 7 \\ 7 & 5 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 2 & 9 \end{bmatrix}$$

Then A+B = $\begin{bmatrix} 5 & 6 & 7 \\ 8 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 7 \\ 7 & 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 9 & 7 & 14 \\ 15 & 7 & 13 \end{bmatrix}$$

Definition

Let A = $[a_{ij}]$ be an m x n matrix and B = $[b_{ij}]$ be an n x r matrix. The product AB is a matrix C = $[c_{ij}]$ of order m x r with

$$C_{ij} = \sum_{k}^{n} = 1\, a_{ik}b_{kj} = a_{i1}\, b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}.$$

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 3 \\ 1 & 0 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 3 \\ 1 & 0 & 4 \end{bmatrix}$$

Definition

The matrix A is said to be row matrix if A is of order Ixn.

Example

A= [1 0 9]

Definition

The matrix A is said to be column matrix if A is of order n x I.

Example

The matrix A is said to be column matrix if A is of order n x I.

Example

$$A = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Definition

The operation + is defined on [0,1] by

A+b= max (a,b),

Where a,bε [0,1].

Example

Let X = {0, 1,2,3} be the set.

The fuzzy sets are defined by

$$\mu_1(X) = \begin{cases} 0.1 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \end{cases}$$

$$\mu_2(x) = \begin{cases} 0.6 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \end{cases}$$

Then $\mu_1(x)+\mu_2(x)$ = max $(\mu_1(x)+\mu_2(x))$

$$= \begin{cases} 0.6 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.2 & \text{if } x = 3 \end{cases}$$

Definition

The operation is defined on [0,1] by

a.b = min (a,b),

where a, b ε [0,1].

Example

Let X = {0, 1,2,3} be the set.

The fuzzy sets are defined by

$$\mu_1(X) = \begin{cases} 0.1 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \end{cases}$$

$$\mu_2(x) = \begin{cases} 0.6 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \end{cases}$$

Then $\mu_1(x).\mu_2(x)$ = min $(\mu_1(x)+\mu_2(x))$

$$= \begin{cases} 0.1 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.2 & \text{if } x = 3 \end{cases}$$

Definition

The operation ← is defined on [0, 1] by

$$a \leftarrow b = \begin{cases} 0 & \text{if } a > b \\ a & \text{if } a \le b \end{cases}$$

Where a, b ε [0,1].

Example

Let X = {0, 1,2,3} be the set.

The fuzzy sets are defined by

$$\mu_1(X) = \begin{cases} 0.1 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \end{cases}$$

$$\mu_2(x) = \begin{cases} 0.6 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \end{cases}$$

Then $\mu_1(x) \leftarrow \mu_2(x) = \begin{cases} 0 & \text{if } \mu_1 > \mu_2 \\ a & \text{if } \mu_1 \le \mu_2 \end{cases}$

$$= \begin{cases} 0.1 & \text{if } x = 0,1 \\ 0.5 & \text{if } x = 2 \\ 0.2 & \text{if } x = 3 \end{cases}$$

Definition

The operation - is defined on [0, 1] by

$$a \leftarrow b = \begin{cases} a & \text{if } a > b \\ 0 & \text{if } a \le b \end{cases}$$

Where a, b ε [0,1].

Example

Let X = {0, 1,2,3} be the set.

The fuzzy sets are defined by

$$\mu_1(X) = \begin{cases} 0.1 & \text{if} x = 0,1 \\ 0.5 & \text{if} x = 2 \\ 0.3 & \text{if} x = 3 \end{cases}$$

$$\mu_2(x) = \begin{cases} 0.6 & \text{if} x = 0,1 \\ 0.5 & \text{if} x = 2 \\ 0.3 & \text{if} x = 3 \end{cases}$$

Then $\mu_1(x) \leftarrow \mu_2(x) = \begin{cases} \mu_1 & \text{if} \mu_1 > \mu_2 \\ 0 & \text{if} \mu_1 \leq \mu_2 \end{cases}$

$$= \begin{cases} 0 & \text{if} = 0,1 \\ 0.5 & \text{if} x = 2 \\ 0.3 & \text{if} x = 3 \end{cases}$$

Representation of Fuzzy Matrices based on Reference Function

In this chapter we discuss about the representation of fuzzy matrices based on reference function and finding trace of such matrices. This chapter is divided into two sections first section deals with representation of fuzzy matrices based on reference function and its operation and second deals with finding the trace of such matrices.

Fuzzy Matrices Based on Reference Function and its Operations

In this section, we deal about representation of fuzzy matrices based on reference function and its operation.

Definition

Let A = (μ_{ij}) be the given fuzzy matrix. Then A can be represented using the reference function by

A = $(\mu_{ij}, 0)$

Or

A = $(\mu_{ij}, 0)$ for all i,j

Example

$$A = \begin{bmatrix} (0.3-0) & (0.7-0) & (0.8-0) \\ (0.4-0) & (0.5-0) & (0.3-0) \\ (0.6-0) & (0.1-0) & (0.4-0) \end{bmatrix}$$

$$A = \begin{bmatrix} (0.3,0) & (0.7,0) & (0.8,0) \\ (0.4,0) & (0.5,0) & (0.3,0) \\ (0.6,0) & (0.1,0) & (0.4,0) \end{bmatrix}$$

Definition

Let A be the given fuzzy matrix represented by reference function. Then complement of A is given by

$A^c = (1, \mu_{ij})$

or

$A^c = (1-\mu_{ij})$ for all i,j.

Example

$$A^c = \begin{bmatrix} (0.3-0) & (0.7-0) & (0.8-0) \\ (0.4-0) & (0.5-0) & (0.3-0) \\ (0.6-0) & (0.1-0) & (0.4-0) \end{bmatrix}$$

Then A^c is given by

or

$$A^c = \begin{bmatrix} (1,0.3) & (1,0.7) & (1,0.8) \\ (1,0.4) & (1,0.5) & (1,0.3) \\ (1,0.6) & (1,0.1) & (1,0.4) \end{bmatrix}$$

Definition

Let A (μ_{ij}) and B = (σ_{ij}) be two fuzzy matrices represented using reference function of order m x n. Then addition two matrices is defined by

A + B = {max (μ_{ij}, σ_{ij}), min $(\gamma_{ij}, \gamma^{*}_{ij})$}

Where

μ_{ij} is membership values of matrix A and γ_{ij} represents corresponding reference function.

σ_{ij} is membership values of matrix B and γ'_{ij} represents corresponding reference function.

Example

$$A = (\mu_{ij}) = \begin{bmatrix} (0.3-0) & (0.7-0) & (0.8-0) \\ (0.4-0) & (0.5-0) & (0.3-0) \\ (0.6-0) & (0.1-0) & (0.4-0) \end{bmatrix}$$

$$B = (\sigma_{ij}) = \begin{bmatrix} (1-0) & (0.2-0) & (0.3-0) \\ (0.8-0) & (0.5-0) & (0.2-0) \\ (0.5-0) & (1-0) & (0.8-0) \end{bmatrix}$$

A+B = $\mathbb{C}$ = (β_{ij})

β_{11} = {max (μ_{11}, σ_{11}), min $(\gamma_{11}, \gamma'_{11})$}

= {max (0.3, 1), min(0,0)}

= (1,0)

$\beta_{12} = \{\max(\mu_{12},\sigma_{12}), \min(\gamma_{12}, \gamma'_{12})\}$
$= \{\max(0.7, 0.2), \min(0,0)\}$
$= (0.7, 0)$

$\beta_{13} = \{\max(\mu_{13},\sigma_{13}), \min(\gamma_{13}, \gamma'_{13})\}$
$= \{\max(0.8, 0.3), \min(0,0)\}$
$= (0.8, 0)$

$\beta_{21} = \{\max(\mu_{21},\sigma_{21}), \min(\gamma_{21}, \gamma'_{21})\}$
$= \{\max(0.4, 0.8), \min(0,0)\}$
$= (0.8, 0)$

$\beta_{22} = \{\max(\mu_{22},\sigma_{22}), \min(\gamma_{22}, \gamma'_{22})\}$
$= \{\max(0.5, 0.5), \min(0,0)\}$
$= (0.5, 0)$

$\beta_{23} = \{\max(\mu_{23},\sigma_{23}), \min(\gamma_{23}, \gamma'_{23})\}$
$= \{\max(0.3, 0.2), \min(0,0)\}$
$= (0.3, 0)$

$\beta_{31} = \{\max(\mu_{31},\sigma_{31}), \min(\gamma_{31}, \gamma'_{31})\}$
$= \{\max(0.5, 0.6), \min(0,0)\}$
$= (0.6, 0)$

$\beta_{32} = \{\max(\mu_{32},\sigma_{32}), \min(\gamma_{32}, \gamma'_{32})\}$
$= \{\max(0.1, 1), \min(0,0)\}$
$= (1, 0)$

$\beta_{33} = \{\max(\mu_{33},\sigma_{33}), \min(\gamma_{33}, \gamma'_{33})\}$
$= \{\max(0.4, 0.8), \min(0,0)\}$
$= (0.8, 0)$

$$A + B = \begin{bmatrix} (1-0) & (0.2-0) & (0.3-0) \\ (0.8-0) & (0.5-0) & (0.2-0) \\ (0.5-0) & (1-0) & (0.8-0) \end{bmatrix}$$

Example:

$A^c = (\mu_{ij})$

$$= \begin{bmatrix} (1-0.3) & (1-0.7) & (1-0.8) \\ (1-0.4) & (1-0.5) & (1-0.3) \\ (1-0.6) & (1-0.1) & (1-0.4) \end{bmatrix}$$

$B^c = (\sigma_{ij}) =$

$$\begin{bmatrix} (1-1) & (1-0.2) & (1-0.3) \\ (1-0.8) & (1-0.5) & (1-0.2) \\ (1-0.5) & (1-1) & (1-0.8) \end{bmatrix}$$

$A^c + B^c = \mathcal{C}^c = (\beta_{ij})$

$\beta_{11} = \{\max(\mu_{11},\sigma_{11}), \min(\gamma_{11}, \gamma'_{11})\}$
$= \{\max(0.3, 1), \min(1,1)\}$
$= (1, 1)$

$\beta_{12} = \{\max(\mu_{12},\sigma_{12}), \min(\gamma_{12}, \gamma'_{12})\}$
$= \{\max(0.7, 0.2), \min(1,1)\}$
$= (1, 0.7)$

$\beta_{13} = \{\max(\mu_{13},\sigma_{13}), \min(\gamma_{13}, \gamma'_{13})\}$
$= \{\max(0.8, 0.3), \min(1,1)\}$
$= (1, 0.8)$

$\beta_{21} = \{\max(\mu_{21},\sigma_{21}), \min(\gamma_{21}, \gamma'_{21})\}$
$= \{\max(0.4, 0.8), \min(1,1)\}$
$= (1, 0.8)$

$\beta_{22} = \{\max(\mu_{22},\sigma_{22}), \min(\gamma_{22}, \gamma'_{22})\}$
$= \{\max(0.5, 0.5), \min(1,1)\}$
$= (1, 0.5)$

$\beta_{23} = \{\max(\mu_{23},\sigma_{23}), \min(\gamma_{23}, \gamma'_{23})\}$
$= \{\max(0.3, 0.2), \min(1,1)\}$
$= (1, 0.3)$

$\beta_{31} = \{\max(\mu_{31},\sigma_{31}), \min(\gamma_{31}, \gamma'_{31})\}$
$= \{\max(0.5, 0.6), \min(0,0)\}$
$= (1, 0.6)$

$\beta_{32} = \{\max(\mu_{32},\sigma_{32}), \min(\gamma_{32}, \gamma'_{32})\}$
$= \{\max(0.1, 1), \min(1,1)\}$
$= (1, 1)$

$\beta_{33} = \{\max(\mu_{33},\sigma_{33}), \min(\gamma_{33}, \gamma'_{33})\}$
$= \{\max(0.4, 0.8), \min(1,1)\}$
$= (1, 0.8)$

$$A + B = \begin{bmatrix} (1-1) & (1-0.7) & (1-0.8) \\ (1-0.8) & (1-0.5) & (1-0.3) \\ (1-0.6) & (1-1) & (1-0.8) \end{bmatrix}$$

Definition

Let $A = (\mu_{ij})$ and $B = (\sigma_{ij})$ be two fuzzy matrices represented using reference function of order m x n and n x m respectively. Then multiplication two matrices is defined by

$A \,.\, B = \{\max\min(\mu_{ij},\sigma_{ij}), \min\max(\gamma_{ij}, \gamma'_{ij})\}$

Example

$A = (\mu_{ij}) = \begin{bmatrix} (0.1-0) & (0.3-0) \\ (0.5-0) & (0.7-0) \end{bmatrix}$

$B = (\sigma_{ij}) = \gamma'_{ij}$

$A.B = \begin{bmatrix} (0.1-0) & (0.3-0) \\ (0.5-0) & (0.7-0) \end{bmatrix}$

$\begin{bmatrix} (0.5-0) & (0.2-0) \\ (0.7-0) & (0.8-0) \end{bmatrix}$

$= (\beta_{ij})$

β_{11} = {max min (μ_{11},σ_{11}), min max $(\gamma_{11}, \gamma'_{11})$}

= {max [min (0.1,0.5), min(0.3,0.7)], min [max (0,0), max (0,0)}

= (0.3,0)

β_{12} = {max min (μ_{12},σ_{12}), min max $(\gamma_{12}, \gamma'_{12})$}

= {max [min (0.1,0.2), min(0.3,0.8)], min [max (0,0), max (0,0)}

= (0.3,0)

β_{21} = {max min (μ_{21},σ_{21}), min max $(\gamma_{21}, \gamma'_{21})$}

= {max [min (0.5,0.5), min(0.7,0.7)], min [max (0,0), max (0,0)}

= (0.7,0)

β_{22} = {max min (μ_{11},σ_{11}), min max $(\gamma_{11}, \gamma'_{11})$}

= {max [min (0.5,0.5), min(0.7,0.8)], min [max (0,0), max (0,0)}

= (0.7,0)

$A.B = \begin{bmatrix} (0.3-0) & (0.3-0) \\ (0.7-0) & (0.7-0) \end{bmatrix}$

Example

$A = \begin{bmatrix} (0.1-0) & (0.3-0) \\ (0.5-0) & (0.7-0) \end{bmatrix}$

$B = \begin{bmatrix} (0.5-0) & (0.2-0) \\ (0.7-0) & (0.8-0) \end{bmatrix}$

Then

$A^c = (\mu_{ij}) = \begin{bmatrix} (1-0.1) & (1-0.3) \\ (1-0.5) & (1-0.7) \end{bmatrix}$

$B^c = (\sigma_{ij}) = \begin{bmatrix} (1-0.5) & (1-0.2) \\ (1-0.7) & (1-0.8) \end{bmatrix}$

$A^c.B^c = \begin{bmatrix} (1-0.1) & (1-0.3) \\ (1-0.5) & (1-0.7) \end{bmatrix}$.

$\begin{bmatrix} (1-0.5) & (1-0.2) \\ (1-0.7) & (1-0.8) \end{bmatrix}$

$= \begin{bmatrix} (1-0.5) & (1-0.2) \\ (1-0.5) & (1-0.5) \end{bmatrix}$

Example

Let A, B, $\mathbb{C}$ be three fuzzy matrices of order m x n, n x p, p x q respectively, then multiplication of fuzzy matrices is associative if conformability is assured

$A.(B\mathbb{C}) = (AB).\mathbb{C}$

The same result hold for complementation fuzzy matrices

$A^c.(B^c.\mathbb{C}^c) = (A^c.B^c).\mathbb{C}^c$.

Example

$A = \begin{bmatrix} (0.1-0) & (0.3-0) \\ (0.5-0) & (0.7-0) \end{bmatrix}$

$B = \begin{bmatrix} (0.5-0) & (0.2-0) \\ (0.7-0) & (0.8-0) \end{bmatrix}$

$\mathbb{C} = \begin{bmatrix} (0.1-0) & (0.3-0) \\ (0.5-0) & (0.7-0) \end{bmatrix}$

Then

$A^c = \begin{bmatrix} (1-0.1) & (1-0.3) \\ (1-0.5) & (1-0.7) \end{bmatrix}$

$B^c = \begin{bmatrix} (1-0.5) & (1-0.3) \\ (1-0.7) & (1-0.7) \end{bmatrix}$

$\mathbb{C}^c = \begin{bmatrix} (1-0.1) & (1-1) \\ (1-0.9) & (1-0.6) \end{bmatrix}$

Then

$A^c.B^c =$

$\begin{bmatrix} (1-0.5) & (1-0.2) \\ (1-0.5) & (1-0.5) \end{bmatrix}$

$A^c.(B^c.\mathbb{C}^c) = \begin{bmatrix}(1-0.5) & (1-0.6)\\(1-0.5) & (1-0.6)\end{bmatrix}$

$(A^c.B^c)\mathbb{C}^c = \begin{bmatrix}(1-0.5) & (1-0.6)\\(1-0.5) & (1-0.6)\end{bmatrix}$

Therefore, $A^c.(B^c.\mathbb{C}^c) = (A^c.B^c)\mathbb{C}^c$

Properties

Let A, B , $\mathbb{C}$ be three fuzzy matrices of order m x n, n x p, n x q respectively. Then multiplication of fuzzy matrices in distributive with respect to addition of fuzzy matrices.

$A.(B+\mathbb{C}) = A.B + A.\mathbb{C}$

The same result hold for complementation fuzzy matrices

$A^c.(B^c+\mathbb{C}^c) = A^c.B^c + A^c.\mathbb{C}^c$

Properties

Let A, B , $\mathbb{C}$ be three fuzzy matrices of order m x n, n x p, n x q respectively, then multiplication of fuzzy matrices is not always commutative.

$A.B \neq B.A$

The same result hold for complementation fuzzy matrices

$A^c.B^c \neq B^c.A^c$

Example

$A = \begin{bmatrix}(0.1-0) & (0.3-0)\\(0.5-0) & (0.7-0)\end{bmatrix}$

$B = \begin{bmatrix}(0.5-0) & (0.2-0)\\(0.7-0) & (0.8-0)\end{bmatrix}$

Then

$A^c.B^c = \begin{bmatrix}(1-0.5) & (1-0.2)\\(1-0.5) & (1-0.5)\end{bmatrix}$

$B^c.A^c = \begin{bmatrix}(1-0.5) & (1-0.5)\\(1-0.7) & (1-0.7)\end{bmatrix}$

Therefore, $A^c.B^c \neq B^c.A^c$

Trace of a Fuzzy Matrix

In this section, we discuss about the trace of fuzzy matrix.

Definition

Let $A = (\mu_{ij})$ be square matrix of order n. Then the trace of matrix A is denoted by trA and is defined by

$trA = (\max \mu_{ij} \min \gamma_{ij})$.

Example

$A = (\mu_{ij})$ = Then trace A is given by

$trA = (\max \mu_{ij}, \min \gamma_{ij})$

$= (\max(0.3, 0.5, 0.4), \min(0,0,0))$

$= (0.5,0)$

Theorem

Let Aand B be two fuzzy square matrices each of order n and λ be any scalar such that $0 \leq \lambda \leq 1$, then

(i) $tr(A+B) = trA + trB$

(ii) $tr(\lambda A) = \lambda tr(A)$.

(iii) $trA = trA'$. where A is the transpose of A.

Proof

Let $A = (\mu_{ij})$ and $B = (\sigma_{ij})$be two fuzzy matrices of order n.

$trA = \{\max(\mu_{ij}), \min(\sigma_{ij})\}$ and

$trB = \{\max(\sigma_{ij}), \min(\gamma'_{ij})\}$ and

Then, $A + B = \mathbb{C}$, where $\mathbb{C} = [\beta_{ij}]$

$\beta_{ij} = \{\max(\mu_{ij},\sigma_{ij}), \min(\gamma_{ij}, \gamma'_{ij})$

By definition of trace of a fuzzy matrix, we have

$tr\,\mathbb{C} = [\max\{\max(\mu_{ij},\sigma_{ij})\}, \min\{\min(\gamma_{ij}, \gamma'_{ij})\}]$

$= [\max\{\max(\mu_{ij}), \max(\sigma_{ij})\}, \min\{\min((\gamma_{ij}), \min(\gamma'_{ij})\}]$

$= trA + trB$

Conversely,

$trA + trB = [\max\{\max(\mu_{ij}), \max(\sigma_{ij})\}, \min\{\min((\gamma_{ij}), \min(\gamma'_{ij})\}]$

$= [\max\{\max(\mu_{ij}, \sigma_{ij})\}, \min\{\min((\gamma_{ij}, \gamma'_{ij})\}]$

$= tr\,\mathbb{C}$

Therefore, trA + trB= tr $\mathbb{C}$

(ii) tr (λA) = {max ($\lambda\mu_{ij}$), min ($\lambda\gamma_{ij}$)

= λ {max (μ_{ij}), min (γ_{ij})}

= λtr(A)

Therefore, tr(λ) = λtr(A).

(iii) trA = {max (μ_{ij}), min (γ_{ij})}

Let A' be the transpose of A.

Then, trA = {max(μ_{ij}), min (γ_{ij})}

Therefore, trA = trA'

Bibliography

1. Baruah H K, Puzzy Membership with respect to a Reference Function, Journal of the Assam Science Society, 1999, 40(.3) : 65-73.
2. Baruah H K. Towards Forming A Field of Fuzzy Sets, International Journal of Energy Information and Communications, 2011, 2(1) : 16-20.
3. Baruah H K, Theroy of Fuzzy sets Beliefs and Realities, International Journal of Energy, Information and Communications, 2011, 2(2) : 1-22.
4. Kandasamy W.B., Elementary Fuzzy Matrix Theory and Fuzzy Models, Automation, 2007.
5. Kaufmann, A., Introduction to the Theory of Fuzzy Subjects, Academic Press, New York (1975).
6. Kim, K.H. and Roush, F.W., Generalized fuzzy matrices. Fuzzy Sets and Systems 4 : 293 – 315 (1980).
7. MamoniDhar, Representation of Fuzzy Matrices Based on References Function, I.J. Intelligent Systems and Applications, 2013, 02, 84 – 90.
8. Sanchex, E., Resolution of composite fuzzy relation equations, inform and Control 30 : 38 – 48 (1976).
9. F.I.Sidky and E.G. Emam, Some Remarks on Section of a Fuzzy Matrix.