A Distance Measure Between Intuitionistic Fuzzy Multisets Sets in Pattern Recognition

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Abstract

In this paper, we propose new method to calculate the distance between intuitionistic fuzzy sets (IFSs) based on the three dimensional representation of IFSs and analyze the relations of similarity measure and distance measure of IFSs. Finally, we apply the proposed measures to pattern recognitions.

Keywords: Intuitionistic Fuzzy Sets, Distance Measures, Similarity Measures, Pattern Recognition

Introduction

Fuzzy set theory, proposed by Zadeh[1] in 1965.The concept of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov [2]. In [3] Shinoj T.K and Sunil Jacob John have discussed Intuitionistic fuzzy Multisets and its application in medical diagnosis.These IFSs have been widely studied in [4,5,6,7] and applied for measuring distances between Intuitionistic Fuzzy Sets. In [8,9,10,11,12] have discussed similarity measures between IFSs. I.K. Vlachos, G.D. Sergiadis [14] and Z. Liang, P. Shi[13] have showed how these measures may be used in pattern recognition problems.[15], Jin Han Park1, Ki Moon Lim1 and Young ChelKwun have derived a Distance measure between intuitionistic fuzzy sets and discuused its application to pattern recognition. A.Maheswari, S.Karthikeyan, P.Veerammal and M.Palanivelrajan [16] have derived a new distance measure between Intuitionistic Fuzzy Multisets sets and also used this distance measure in Medical Diagnosis. In this paper, we calculate the distance between IFMs based on the three dimensional representation of IFMs and find the similarity measure and distance measure of IFMs, also, we apply this measures in pattern recognitions.

Preliminaries

In this section, we recall some relevant definitions and results. Throughout this paper, X is a universal set.

Definition 2.1 A Fuzzy Set (FS) A $\in$ X is defined as an object of the form

A = {<x, μ_A<(x) >/ x $\in$ X}, where the function μ_A: X→[0,1] denote the degree of membership function of A.

Definition 2.2 A Fuzzy multiset (FMS) A$\in$X is defined as an object of the form

A = {< x, (μ^1_A(x), μ^2_A (x), ... μ^n_A(x))>/ x $\in$X}, where the function

μ^i_A (x): X→[0,1], $1 \le i \le n$ with $\mu^1_A(x) \ge \mu^2_A(x) \ge \ldots \ge \mu^n_A(x)$.

Definition 2.3 An Intuitionistic Fuzzy Set (IFS) A X is defined as an object of the form

A = {< x , μ_A(x), ν_A (x) X >/x $\in$ X}, where the function μ_A: X→[0,1] and ν_A: X→[0,1] denote the degree of membership and the degree of non-membership function of A respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$, for every x in X. To measure hesitancy degree of an element to an IFS, A tan as so v introduced a function given by, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, ∀ and $0 \le \pi_A(x) \le 1$

Definition 2.4 An Intuitionistic Fuzzy Multiset (IFMs) A $\in$ X is defined as an object of the form

A= {< x, (μ^1_A(x), μ^2_A (x), ... μ^n_A(x)), (v^1_A (x), v^2_A (x), ... v^n_A(x) >/ x $\in$X}, where the function μ^i_A (x) : X→[0,1], $1 \le i \le n$ with $\mu^1_A(x) \ge \mu^2_A(x) \ge \ldots \ge \mu^n_A(x)$ and v^i_A(x): X→[0,1], $1 \le i \le n$ denote the degree of membership and the degree of non-membership function of A respectively and $0 \le \mu^i_A(x) + v^i_A(x) \le 1$, for every x in X.

Definition 2.5 An Intuitionistic Fuzzy Multisets (IFMs) A, B $\in$ X are defined as an object of the form

A = {< x, (μ^1_A(x), μ^2_A (x), ... μ^n_A (x)), (v^1_A(x), v^2_A(x), ... v^n_A(x)) >/x $\in$ X}and

B = {< x, (μ^1_A(x), μ^2_A (x), ... μ^n_A (x)), (v^1_A(x), v^2_A(x), ... v^n_A(x)) >/x $\in$ X}

Definition 2.6 Let X = {x_1, x_2, … x_n} be finite universe of discourse. For any A and B $\in$ IFS s(X), the operation is defined: A = Bi $\mu_A = \mu_B$ and $v_A = v_B$.

Definition 2.7 A distance metric d in a nonempty set X is a real function: X×X→[0, +∞), which satisfies the following conditions:

- d(x,y) $\ge$ 0, for all x,y $\in$ X
- d(x,y) = 0 ⇔ x = y, for all x,y$\in$X
- d(x,y) = d(y,x), for all x,y$\in$X
- d(x,y) + d(z,y) $\ge$ = d(y,x), for all x,y$\in$X

Definition 2.8 A function S: IFS s(X) × IFS s(X) → [0, 1] is called similarity measure of IFSs if S satisfies the following properties: for any A, B, C, $\in$ IFSs(X),

(SP1) $0 \le$ S(A,B) ≤ 1

(SP2) S(A,B) = 1 if and only if A = B

(SP3) S(A,B) = S(B,A)

(SP4) If A $\subseteq$ B $\subseteq$ C, then S(A,C) $\le$ S(A,B) and S(A,C) $\le$ S(B,C)

Definition 2.9 A function d: IFSs(X) × IFSs(X) → [0, 1] is called distance measure of IFSs if d satisfies the following properties: for any A, B, C $\in$ IFSs(X),

- (DP1) $0 \le$ d(A,B) ≤ 1
- (DP2) d(A,B) = 0 if and only if A = B
- (DP3) d(A,B) = d(B,A)
- (DP4) If A $\subseteq$ B $\subseteq$ C, then d(A,C) $\ge$ d(A,B) and d(A,C) $\ge$ d(B,C)

Distance Measures between IFMs

Definition 3.1 Given a finite universe of discourse X = {x_1, x_2, …, x_n}. Let A, B be two intuitionistic fuzzy multisets in IFMs (X), f: [0, 1] [0, 1] → [0, 1] a strictly increasing (or decreasing) binary function for each argument. A distance measure is a function $_{pv}$: IFMs (X) IFMs(X) → [0, 1] defined for A, B $\in$ IFM(X) by d_{pv} (A,B) = 1/4, $\int_0^1 \sum_{i=1}^n \{|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| \lambda + |\pi_A(x_i) - \pi_B(x_i)| \lambda^2\} d\lambda$, Where $\lambda \in [0, 1]$.

Remark 3.2 Clearly, d_{pv} (A,B) is a metric.

Definition 3.3 A function d_{pv}: IFMs (X) × IFMs(X) → [0, 1] is called distance measure of IFMs if d satisfies the following properties, for any A,B,C ∈ IFMs(X),

- $0 \leq d_{pv}(A,B) \leq 1$
- $d_{pv}(A,B) = 0$ if and only if $A = B$
- $d_{pv}(A,B) = d_{pv}(B,A)$
- If $A \subseteq B \subseteq C$, then $d_{pv}(A,C) \geq d_{pv}(A,B)$ and $d_{pv}(A,C) \geq d_{pv}(B,C)$

Definition 3.4 A Similarity distance measures between A, B defined by $S_{PV}(A,B) = 1 - d_{pv}(A,B)$

Definition 3.5 Assume the weight of the element $X = \{x_1, x_2, \ldots, x_n\}$ is w_i, where $0 \leq w_i \leq 1$

Let A, B be two intuitionistic fuzzy multisets in IFMs (X). A weighted distance measures between IFMs is defined by

$d^w_{pv}(A,B) = 1/4 \int^1_0 w_i (\sum^n_{i=2}\{| \mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| \lambda + | \pi_A(x_i) - \pi_B(x_i)| \lambda^2\})d\lambda / \sum^n_{i-1} w^i$, Where $\lambda \in [0, 1]$.

Theorem 3.6 d_{pv} is the degree of distance between two IFMs A and B in $X = \{x_1, x_2, x_3 .... x_n\}$

Proof It is easy to see that d_{pv} satisfies the first three properties of **Definition 3.3**. It is enough to prove that satisfies the fourth property **Definition 3.3**. Let $A \subseteq B \subseteq C$ where A,B,C ∈ IFMs(X). Then $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$; $v_A(x_i) \leq v_B(x_i) \leq v_C(x_i)$ and $\pi_A(x_i) \geq \pi_B(x_i) \geq \pi_C(x_i)$, for any $x_i \in X$. It follows that

$|\mu_A(x_i) - \mu_C(x_i)| \geq | \mu_A(x_i) - \mu_B(x_i)| \geq , |v_A(x_i) - v_C(x_i)| > | v_A(x_i)|, | \pi_A(x_i) - \pi_C(x_i)| > | \pi_A(x_i) - \pi_B(x_i)|$

So, we have

$| \mu_A(x_i) - \mu_C(x_i)| + | v_A(x_i) - v_C(x_i)| \geq \lambda + | \pi_A(x_i) - \pi_C(x_i)| \lambda^2 \geq | \mu_A(x_i) - \mu_C(x_i)| + | v_A(x_i) - v_C(x_i)| \geq \lambda + | \pi_A(x_i) - \pi_C(x_i)| \lambda^2 \Rightarrow 1/4 \int^1_0 \sum^n_{i=2}\{| \mu_A(x_i) - \mu_C(x_i)| + |v_A(x_i) - v_B(x_i)| \lambda + | \pi_A(x_i) - \pi_C(x_i)| \lambda^2\})d\lambda \geq 1/4 \int^1_0 \sum^n_{i=2}\{| \mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| \lambda + | \pi_A(x_i) - \pi_B(x_i)| \lambda^2\})d\lambda \Rightarrow d_{pv}(A,C) \geq d_{pv}(A,B)$. Similarly, we can get $d_{pv}(A,B) \geq d_{pv}(B,C)$.

Theorem 3.7 $d^w_{pv}(A,B)$ is the degree of distance between two IFMs A and B in $X = \{x_i, x_2, x_3 .... x_n\}$

Example 3.8 Let A and B be two IFMs in $X = \{x_1, x_2, x_3\}$ given by $A = \{(x_1, 0.3, 0.6, 0.1), (x_2, 0.5, 0.4, 0.1), (x_3, 0.7, 0.1, 0.2)\}$ and $B = \{(x_1, 0.4, 0.6, 0), (x_2, 0.6, 0.3, 0.1), (x_3, 0.5, 0.2, 0.3)\}$. Then by the definition of $d_{pv}(A,B) = 1/4 \int^1_0 \sum^n_{i=2}\{| \mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| \lambda + | \pi_A(x_i) - \pi_B(x_i)| \lambda^2\})d\lambda = 0.1415$

Example 3.9 Let $X = \{x_1, x_2, x_3\}$are any three patterns given by $A_1 = \{(x_1, 0.3, 0.3, 0.4), (x_2, 0.2, 0.2, 0.6), (x_3, 0.1, 0.1, 0.8)\}$; $A_2 = \{(x_1, 0.2, 0.2, 0.6), (x_2, 0.2, 0.2, 0.6), (x_3, 0.2, 0.2, 0.6)\}$ and $B = \{(x_1, 0.3, 0.3, 0.4), (x_2, 0.2, 0.2, 0.6), (x_3, 0.1, 0.1, 0.8)\}$.

Then the corresponding similarity measures between A and B are given by $S_{pv}(A_1,B) = 1 - d_{pv}(A_1,B) = 1.000$; $S_{pv}(A_2,B) = 1 - d_{pv}(A_2,B) = 1$ 0.893.

Results of the similarity measures between A and B have been summarized in the following Table 1. Based on that table, it is seen that the sample B belongs to the pattern A_1 according to the principle of the maximum degree of similarity between IFMs.

Table 1

	(A_1,B)	(A_2,B)	Maximum degree of similarity
Spv (A_i,B)	1.000	0.893	1.000
Conclusion	B belongs to the Pattern A_1		

Example 3.10 Let $X = \{x_1, x_2, x_3\}$ are any three patterns given by $A_1 = \{(x_1, 0.3, 0.3, 0.4), (x_2, 0.2, 0.2, 0.6), (x_3, 0.1, 0.1, 0.8)\}$; $A_2 = \{(x_1, 0.2, 0.2, 0.6), (x_2, 0.2, 0.2, 0.6), (x_3, 0.2, 0.2, 0.6)\}$;

A_3 = {(x_1, 0.4, 0.4, 0.2), (x_2, 0.4, 0.4, 0.2), (x_3, 0.4, 0.4, 0.2)} and B = {(x_1, 0.3, 0.3, 0.4), (x_2, 0.2, 0.2, 0.6), (x_3, 0.1, 0.1, 0.8)}. Given three minerals of mineral fields, each is featured by the content of three minerals and contains one kind of typical hybrid minerals. The three kinds of typical hybrid minerals are represented by IFMs A_1, A_2, A_3 in X respectively. Given another kind of hybrid mineral B, to which field does this kind of mineral B most probably belong to?

Then the corresponding similarity measures between A and B are given by S_{pv} (A_1,B) = 1.000; S_{pv} (A1,B) = 0.893 and S_{pv} (A_3,B) = 0.675. In table-2 we have to compare the similarity measures between A and B with existing similarity measures presented by Jin Han Park1, Ki Moon Lim1 and Young Chel Kwun[15].

Table 2

	(A_1,B)	(A_2,B)	(A_3,B)	maximum degree of similarity
$S_1(A_i,B)$	1.000	0.877	0.600	1.000
$S^2_2(A_i,B)$	1.000	0.847	0.568	1.000
$S_3(A_i,B)$	1.000	0.765	0.429	1.000
$S^2_4(A^i,B)$	1.000	0.719	0.397	1.000
$S_{pv}(A_i,B)$	1.000	0.893	0.675	1.000
Conclusion	B belongs to the pattern A_1			

Based on the above table, we analyze our results is matching with the existing results. Thus the sample B belongs to the pattern A_1 according to the principle of the maximum degree of similarity between IFMs.

Conclusion

Although many distance measures between intuitionistic fuzzy sets and intuitionistic fuzzy multisets have been proposed. In this paper, we introduced the another distance measure between intuitionistic fuzzy multisets and applied it to pattern recognition and the results are compared with the other distance measures.

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